Parity violation in super-dense matter at RHIC

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BNL and Yale
Outline

- Chiral symmetry and parity violation
- From Maxwell to Yang-Mills
- Topology-induced P and CP violation in super-dense matter at RHIC
- P and CP violation and the Early Universe
What is chiral symmetry?
Chiral symmetry: the definition

Greek word: χειρ (cheir) - hand

Lord Kelvin (1893):
“I call any geometrical figure, or groups of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself.”
Parity violation in Nature

Louis Pasteur
1822-1895

Rotation of light polarization in tartaric acid - absent in synthesized one, but present in the one derived from wine lees ↔ different mixtures of left and right crystals
Classical electrodynamics and Maxwell’s equations

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

Faraday-Maxwell induction

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]
Maxwell on heavy ion collisions:

_Gin a body meet a body_  
_Flyin' through the air._  
_Gin a body hit a body,_  
_Will it fly? And where?_  

from “Rigid body songs”,  
by James Clerk Maxwell

dialect translation: "gin" = if
Solution of Maxwell equations in the vacuum

Equation:

\[ \partial_\mu F^{\mu \nu} = 0 \]

Solution:

\[ F^{\mu \nu} = 0 \]
1/2 of all “elementary” particles of the Standard Model are not observable; they are confined within hadrons by “color” interactions.
P and CP invariances are violated by weak interactions

Nobel prize 1957

C.N. Yang

T.D. Lee

Complex CKM mass matrix
Nobel prize 2008

What about strong interactions?
What is QCD?

QCD = Quark Model + Gauge Invariance

local gauge transformation:

$$\psi(x) \rightarrow \exp(i\omega_a(x)t^a) \, \psi(x)$$

$$[t^a, t^b] = i f^{abc} t_c$$

From Maxwell to Yang-Mills
Solution of Yang-Mills equations in the vacuum

Equation:

\[ D^\mu F_{\mu\nu}^a = 0 \]

Solution:

\[ (F_{\mu\nu}^a)^2 = \frac{192\rho^4}{(x^2 + \rho^2)^4} \]

Coupling of space-time and color:

\[ A^a_\mu(x) = \frac{2\eta_{\alpha\mu\nu} x_\nu}{x^2 + \rho^2}, \quad \eta_{\alpha\mu\nu} = \begin{cases} \epsilon_{\alpha\mu\nu} & \mu, \nu = 1, 2, 3, \\ \delta_{\alpha\mu} & \nu = 4, \\ -\delta_{\alpha\nu} & \mu = 4. \end{cases} \]
Instantons and topology

Instantons: classical Euclidean solutions of QCD which map color SU(2) onto the sphere $S_3$; in Minkowski space, describe quantum tunneling between degenerate vacua with different topological Chern-Simons numbers.

\[ \nu = \int_{-\infty}^{+\infty} dt \frac{dQ_5}{dt} = 2N_f q[F] \]

\[ q[F] = \frac{g^2}{32\pi^2} \int d^4 x F_{\alpha}^{\mu\nu} \tilde{F}_{\alpha\mu\nu} \]

As a result, chiral charge is no longer conserved.

\[ Q_5 = \int d^3 x \ K_0 \]
Topological number fluctuations in QCD vacuum
QCD vacuum as a Bloch crystal

\[ \langle \mathcal{O} \rangle = \sum_q f(q) \int_q D[\psi] D[\bar{\psi}] D[A] \exp(iS_{QCD}) \mathcal{O}(\psi, \bar{\psi}, A) \]

\[ f(q_1 + q_2) = f(q_1) f(q_2) \quad \Rightarrow \quad f(q) = \exp(i\theta q) \]

“θ - vacuum”

\[ |\theta\rangle = \sum_q e^{i\theta q} |q\rangle \]

Sum over degenerate vacua:

“quasi-momentum” “coordinate”
Sphaleron transitions at finite energy or temperature

C. Rebbi, http://scv.bu.edu/visualization/gallery
Topology-induced change of chirality

\[ \vec{J} = \vec{T} + \vec{S} \]
Diffusion of Chern-Simons number in hot QCD: numerical lattice simulations

B. Alles, M. D’Elia and A. DiGiacomo,
hep-lat/0004020
Sphaleron rate in the SUSY Yang Mills plasma
Perfect fluid contains fluctuating topological charge

Son, Starinets
Diffusion of Chern-Simons number in QCD: real time lattice simulations


Is there a way to observe topological charge fluctuations in experiment?

Relativistic ions create a strong magnetic field:

Initial **spatial** anisotropy

Final **momentum** anisotropy

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**Reaction plane** ($\Psi_R$)

**$X$** (defines $\Psi_R$)

**$H$**
Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).
### Comparison of magnetic fields

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Earth's magnetic field</td>
<td>0.6 Gauss</td>
</tr>
<tr>
<td>A common, hand-held magnet</td>
<td>100 Gauss</td>
</tr>
<tr>
<td>The strongest steady magnetic fields achieved so far in the laboratory</td>
<td>$4.5 \times 10^5$ Gauss</td>
</tr>
<tr>
<td>The strongest man-made fields ever achieved, if only briefly</td>
<td>$10^7$ Gauss</td>
</tr>
<tr>
<td>Typical surface, polar magnetic fields of radio pulsars</td>
<td>$10^{13}$ Gauss</td>
</tr>
<tr>
<td>Surface field of Magnetars</td>
<td>$10^{15}$ Gauss</td>
</tr>
</tbody>
</table>

http://solomon.as.utexas.edu/~duncan/magnetar.html

Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$
Charge asymmetry w.r.t. reaction plane as a signature of strong P and CP violation

\[ \mathcal{L}_\theta = -m \cos \theta (\bar{u}_L u_R + \bar{u}_R u_L) - i m \sin \theta (\bar{u}_L u_R - \bar{u}_R u_L) \]

The axis of angular momentum is a symmetry axis; “left” means “up” and “right” means “down”

\[ \vec{L} \parallel \vec{H} \parallel \vec{\sigma} \]

\[ \chi^+ \vec{\sigma} (\vec{n} - \vec{n}') \chi \]

\[ \frac{dN_{\pm}}{d\Omega} = \text{const} \ (1 \pm \kappa \cos \Omega) \sin \Omega \]

distribution in the polar angle
Charge asymmetry w.r.t. reaction plane as a signature of strong P violation

Electric dipole moment of QCD matter!

Let all fermions be right-handed, \( Q = N_R - N_L > 0 \) this means the spin is parallel to momentum.

Magnetic field pins down the directions of spins and thus induces an electric current.

DK, '04; DK & Zhitnitsky, '06; DK, McLerran, Warringa, '07; Fukushima, DK, Warringa, '08
“Chiral magnetic effect”: quark model picture

DK, L. McLerran, H. Warringa, 0711.0950

Red arrow - momentum; blue arrow - spin;
In the absence of topological charge no asymmetry between left and right (fig.1); the fluctuation of topological charge (fig.2) in the presence of magnetic field induces electric current (fig.3)
The chiral chemical potential

\[ \mu_R = \mu + \mu_5 \]
\[ \mu_L = \mu - \mu_5 \]

If a system has Chirality, Fermi-surfaces Right- and Left-handed fermions differ.

This can be described by a chiral chemical potential \( \mu_5 \)

Study equilibrium response to Magnetic Field
Computing the induced current

Fukushima, DK, Warringa, ‘08

Chiral chemical potential can be seen as time component axial vector field $\mu_5 = A_0^5$

$$\partial_\mu j^\mu = \frac{e^2}{16\pi^2} (F^\mu_\nu L \tilde{F}^\nu_\mu L - F^\mu_\nu R \tilde{F}^\nu_\mu R)$$

Vector current anomalous with axial vector field

$$j^\mu (x) = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu (x)}$$

Do derivative expansion, anomaly constraint determines prefactors

D’Hoker and Goldstone (’85)

$$J_3 = \frac{e B L^3}{2\pi^2} \mu_5$$

Because current due to anomaly, very solid relation. No correction due to gluons, etc.

The Chiral Magnetic Effect:

QCD anomaly provides chirality

EM anomaly provides current
Chiral induction

Maxwell and Faraday:

\[ \mathcal{E} = -\frac{d\Phi_B}{dt}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  

constant magnetic field cannot generate e.m. current

but:

\[ \mathbf{j}_3 = \frac{eB}{2\pi^2} \mu_5 \]

Even a constant magnetic field generates current in the presence of the chiral charge:

chiral induction
Charge separation = parity violation:

\[ \mathbf{P} : \mathbf{p} \rightarrow -\mathbf{p}; \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} \rightarrow \mathbf{L} \]
Analogy to P violation in weak interactions
Charge asymmetry w.r.t. reaction plane: how to detect it?

A sensitive measure of the asymmetry:

\[ a^k a^m = \langle \sum_{ij} \sin(\varphi_{i}^{k} - \Psi_{R}) \sin(\varphi_{j}^{m} - \Psi_{R}) \rangle \]

Expect \( a^+ a^+ = a^- a^- > 0 \); \( a^+ a^- < 0 \)

S. Voloshin, hep-ph/0406311
Strong CP violation at high T?

Figure 2: Charged particle asymmetry parameters as a function of standard STAR centrality bins selected on the basis of charged particle multiplicity in $|\eta| < 0.5$ region. Points are STAR preliminary data for Au+Au at $\sqrt{s_{NN}} = 62$ GeV: circles are $a^2_\pm$, triangles are $a^2_\mp$ and squares are $a_+a_-$. Black lines are theoretical prediction [1] corresponding to the topological charge $|Q| = 1$.

I. Selyuzhenkov et al., STAR Coll., nucl-ex/0510069; October 25, 2005
Strong P, CP violation at high T?

Charge asymmetry w.r.t. reaction plane, \( \sim - a^k a^m \)

S. Voloshin et al [STAR Coll.], QM’08
This analysis is currently being finalized; talk by Dhevan Gangadharan
Caveat

The observable which we use is sensitive to $|\vec{d}_e|^2$, not $\vec{d}_e$;

while in the absence of P and T violation e.d.m. should vanish identically, our observable is P and T even, and so is affected by statistical fluctuations -

we must be careful
Topology - Induced Parity violation (TIP)
Stars and galaxies that can be observed today were born as a result of the evolution of the universe.

Present time (13.7 billion years since the Big Bang)

The universe began in an endless state

Created from "nothing"
What are the implications for the Early Universe?
What is the origin of cosmic magnetic fields?

Magnetic fields are abundant in the Universe at large scales:

3 µG field in Milky Way;

1-40 µG fields in clusters of galaxies

Is the entire Universe chiral?

e.g. M.Longo, arXiv:0812.3437;
thanks to Bj

Magnetic field in M51:
Polarization of emission
Beck 2000
What is the origin of magnetic fields in the Universe?

Primordial magnetic field (E.Fermi, 1949)?

Dynamo in proto-galaxy? Stars? Galaxy?

Domain walls and vortices associated with the $\theta$ vacua carry magnetic field;

Primordial magnetic field generation at the QCD phase transition?

DK, R.Pisarski, M.Tytgat ‘98;
R.Brandenberger, A.Zhitnitsky’00
CP violation in QCD and cosmic magnetic fields?

Domain walls in the QCD vacuum?

Through the quark loops
\[ \mathbf{E}^a \cdot \mathbf{B}^a \]
couples to
\[ \mathbf{E} \cdot \mathbf{B} \]

May create primordial field; how to make it ordered over galactic scales? axions?
Baryons in the Universe are rare:

1 baryon per ~ 2 billions photons in the Cosmic Microwave Background Radiation

~ 400 photons/cm$^3$

~ 1 proton/m$^3$

(Almost) no visible antimatter - why the asymmetry?
What is the origin of the matter-antimatter asymmetry in the Universe?

1. B violation
2. CP violation
3. Non-equilibrium dynamics

A.D. Sakharov,
JETP Lett. 5 (1967) 24
Baryon asymmetry in the Universe and strong CP violation

1. Generation of Chern-Simons number at the QCD phase transition is analogous to baryon number generation in the electroweak phase transition
   
e.g. V. Kuzmin, V. Rubakov and M. Shaposhnikov, 

2. Strong CP violation can lead to the separation of matter and antimatter in the Universe at the QCD phase transition
   
e.g. R. Brandenberger, I. Halperin and A. Zhitnitsky, 
   hep-ph/9903318  
   DK, A. Zhitnitsky, arXiv 0706.1026
Summary

- Topological structure of QCD vacuum makes P and CP violation possible in strong interactions.
- Even in the absence of a global parity violation, sphaleron transitions can induce P-odd fluctuations.
- In heavy ion collisions this topology-induced parity violation can be observed through the event-by-event charge asymmetries.
- Since charge asymmetry requires separation of quarks by “macroscopic” distance, it is a signature of deconfinement and chiral symmetry restoration.
- What are the implications for the Early Universe?