The Strongly Interacting Glasma

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Outline:

The sQGP Paradigm

Saturation and Coherent Fields in the Hadron Wavefunction

The Formation of the Glasma

Early Time Evolution of the Glasma

Thermalization and Transport

Implications
The Space-Time Evolution of Heavy Ion Collisions

An inevitable consequence of QCD above some energy

Well developed and good quantitative description in eA, pp, pA and AA collisions at energies of HERA, RHIC and LHC

Much excitement about the pA run now taking place at LHC
Color Glass Condensate:
The High Density Gluonic States of a high energy hadron that dominate high energy scattering.

Glasma:
Highly coherent gluon fields arising from the Glasma that turbulently evolve into the thermalized sQGP while making quarks

Thermalized sQGP:
Largely incoherent quark and gluons that are reasonably well thermalized
Saturation and Coherent Fields:

Color Glass Condensate:

High density gluon fields:

\[
\frac{dN}{dyd^2r_Td^2p_T} \sim \frac{1}{\alpha_s}
\]

Parton distributions replaced by ensemble of coherent classical fields
Renormalization group equations for sources of these fields

\[Q^{2}_{sat} >> \Lambda_{QCD}^{2}\]
Collisions of two sheets of colored glass

Sheets get dusted with color electric and color magnetic fields

The initial conditions for a Glasma evolve classically and the classical fields radiate into gluons. Longitudinal momentum is red shifted to zero by longitudinal expansion.

But the classical equations are chaotic:

Small deviations grow exponentially in time.
The Glasma

Typical configuration of a single event just after the collision

Highly coherent colored fields:
Stringlike in longitudinal direction
Stochastic on scale of inverse saturation momentum in transverse direction
Multiplicity fluctuates as negative binomial distribution
Ridge seen in high multiplicity pp in CMS

Now seen in high multiplicity pA in CMS, ALICE ATLAS

For fixed multiplicity cut, pA ridge appears to be stronger than in pp, argues against hydro description

pA and pp require sub-nucleonic scales of transverse fluctuations

Subtracting jet contribution leaves an almost pure dipole, as predicted (ALICE)
The Glasma:

Weak coupling but strongly interacting due to coherence of the fields
In transport or classical equations, the coupling disappears!

Two scales

\[ \Lambda_{coh}(t_{in}) \sim \Lambda_{UV}(t_{in}) \sim Q_{sat} \]

But it takes time to separate the scales and make a thermal distribution

\[ \Lambda_{coh}(t_{therm}) \sim \alpha_s \Lambda_{UV}(t_{therm}) \sim \alpha_s T_{init} \]

How long does it take to thermalize?

Are there Bose-Einstein Condensates formed?

For how long is the system inhomogeneous with longitudinal pressure not equal to transverse?

Can we measure a difference between longitudinal and transverse pressure?

Epplebaum Gelis: Scalar field

Order parameters: Electric and magnetic confinement
In scalar field theory:

- Smallish viscosity to entropy
- Eventual equilibration of longitudinal and transverse pressure
- Longish time for thermalization
- Yang Mills theory with realistic numbers?

What condenses?

The Glasma and turbulent coherent fields are generically a new type of matter:

There may be genuinely new phenomenon associated with electric and magnetic confinement and perhaps superfluidity

Vacuum ~ Turbulent Fluctuations?
The Glasma may be a nearly perfect fluid, even though it is not a thermalized sQGP. It is certainly a sQGP.

Viscous hydro can cope with partial thermalization, and large differences between longitudinal and transverse pressures.

In fact, there is little experimental evidence that complete local equilibrium is reached in nuclear collisions.
RHIC and LHC Data on heavy ions

Very Strong Collective Flow Patterns Consistent with Perfect Fluid Hydrodynamics
Small Viscosity to Entropy

“Jet quenching” => Large Energy Loss / Length

Strongly Interacting QGP Paradigm:
Usual Claim:
A well thermalized Quark Gluon Plasma?
Photons and Dileptons

QGP & Hydro:
Magnitude and shape
Flow not explained

QGP & Hydro
Magnitude and shape not explained
Dilepton have $p_T$ slope of 100 MeV
No agreement on data
Chaos and Turbulence:

CGC field is rapidity independent => occupies restricted range of phase space
Wiggling strings have much bigger classical phase space
A small perturbation that has longitudinal noise grows exponentially

\[ A_{\text{classical}} \sim \frac{1}{g} \]
\[ A_{\text{quantum}} \sim 1 \]

After a time

\[ t \sim \frac{\ln p(1/g)}{Q_{\text{sat}}} \]

system isotropizes,

But it has not thermalized!

At later times a fractional anisotropy between P_L and P_T may develop
Thermalization naively occurs when scattering times are small compared to expansion times. Scattering is characterized by a small interaction strength.

How can the system possibly thermalize, or even strongly interact with itself?

Initial distribution:

\[
\frac{dN}{d^3xd^3p} \sim \frac{Q_{\text{sat}}}{\alpha_s E} F(E/Q_{\text{sat}})
\]

A thermal distribution would be:

\[
\frac{dN}{d^3xd^3p} \sim \frac{1}{e^{E/T} - 1} \sim T/E
\]

Only the low momentum parts of the Bose-Einstein distribution remain:

\[
E \sim Q_{\text{sat}}
\]

“\(T \sim Q_{\text{sat}}/\alpha_s\)”

As dynamics migrates to UV, how do we maintain isotropy driven by infrared modes with a scale of the saturation momentum?
Phase space is initially over-occupied

\[ f_{\text{thermal}} = \frac{1}{e^{(E-\mu)/T} - 1} \]

Chemical potential is at maximum the particle mass

\[ \rho_{\text{max}} \sim T^3 \quad \epsilon_{\text{max}} \sim T^4 \]

\[ \rho_{\text{max}}/\epsilon_{\text{max}}^{3/4} \leq C \]

But for isotropic Glasma distribution

\[ \rho_{\text{max}}/\epsilon_{\text{max}}^{3/4} \leq 1/\alpha_S^{1/4} \]
Where do the particle gluons go?

If inelastic collisions were unimportant, then as the system thermalized, the ratio of the energy density and number density are conserved

\[ f_{\text{thermal}} = \rho_{\text{cond}} \delta^3(p) + \frac{1}{e^{(E-m)/T} - 1} \]

One would form a Bose-Einstein Condensate

Over-occupied phase space => Field coherence in important Interactions can be much stronger that \( g^2 \)

\[ N_{\text{coh}} g^2 \]

Might this be at the heart of the large amount of jet quenching, and strong flow patterns seen at RHIC?

Problem we try to solve:

How does the system evolve from an early time over-occupied distribution to a thermalized distribution

We argue that the system stays strongly interacting with itself during this time due to coherence
First: Kinetic Evolution Dominated by Elastic Collisions in a Non-Expanding Glasma

\[ \partial_t f(p, X) = C_p[f] \]

\[ f(p, X) = \frac{\Lambda_s(t)}{\alpha_s p} g(p/\Lambda(t)) \]

\[ \Lambda_s(t_i) \sim \Lambda(t_i) \sim Q_{sat} \]

Small angle approximation for transport equation:

\[ \frac{\partial f}{\partial t} \bigg|_{coll} \sim \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[ \frac{df}{dp} + \frac{\alpha_s}{\Lambda_s} f(p)(1 + f(p)) \right] \right\} \]

Due to coherence, the collision equation is independent of coupling strength! Equation describes a strongly interacting but weakly coupled QGP

There is a fixed point of this equation corresponding to thermal equilibrium when

\[ T \sim \Lambda \sim \frac{\Lambda_s}{\alpha_s} \]
Estimates of various quantities
(Momentum integrations are all dominated by the hard scale)

\[ n_g \sim \frac{1}{\alpha_s} \Lambda^2 \Lambda_s \]
\[ \epsilon_g \sim \frac{1}{\alpha_s} \Lambda_s \Lambda^3 \]
\[ \frac{\epsilon_g}{n_g} \sim \Lambda \]

\[ n = n_c + n_g \]
\[ \epsilon_c \sim n_c m \sim n_c \sqrt{\Lambda \Lambda_s} \]
\[ m^2 \sim \alpha_s \int dp p^2 \frac{df(p)}{d\omega_p} \sim \Lambda \Lambda_s \]

The collision time follows from the structure of the transport equation and is

\[ t_{scat} = \frac{\Lambda}{\Lambda_s^2} \]

The scattering time is independent of the interaction strength
Thermalization in a non-expanding box

\[ \epsilon = \Lambda_s \Lambda^3 \sim \text{constant} \quad \text{So that} \quad \Lambda_s \sim Q_s \left( \frac{t_0}{t} \right)^{\frac{3}{7}} \]

\[ t_{\text{scat}} = \frac{\Lambda}{\Lambda_s^2} \sim t \]

\[ \Lambda \sim Q_s \left( \frac{t}{t_0} \right)^{\frac{1}{7}} \]

\[ n_g \sim n_0 \left( \frac{t_0}{t} \right)^{\frac{1}{7}} \quad m \sim Q_s (t_0/t)^{1/7} \quad \frac{\epsilon_c}{\epsilon_g} \sim \left( \frac{t_0}{t} \right)^{1/7} \]

\[ s \sim \Lambda^3 \sim Q_s^3 (t/t_0)^{3/7} \]

At thermalization \( \Lambda_s = \alpha_s \Lambda \)

\[ t_{\text{th}} \sim \frac{1}{Q_s} \left( \frac{1}{\alpha_s} \right)^{\frac{7}{4}} \quad s \sim \frac{Q_s^3}{\alpha_s^{3/4}} \sim T^3 \]
How do inelastic processes change this?

Rates of inelastic and elastic processes are parametrically the same.

What about the condensate? Difficult to make definite statement.

In relaxation time limit, we would expect:

\[
\frac{1}{t_{\text{scat}}} \sim \alpha_s^{n+m-2} \left( \frac{\Lambda_s}{\alpha_s} \right)^{n+m-2} \left( \frac{1}{m^2} \right)^{n+m-4} \Lambda^{n+m-5}
\]

\[
m^2 \sim \Lambda_s \Lambda
\]

\[
t_{\text{scat}} = \frac{\Lambda}{\Lambda_s^2},
\]

What about the condensate? Difficult to make definite statement.

In relaxation time limit, we would expect:

\[
\frac{d}{dt} \rho_{\text{cond}} = -\frac{a}{t_{\text{scat}}} \rho_{\text{cond}} + \frac{b}{t_{\text{scat}}} n_{\text{gluons}}
\]

Either \( \rho_{\text{cond}} \gg n_{\text{gluons}} \) or \( \rho_{\text{cond}} = \frac{b}{a} n_{\text{gluons}} \)

Condensate would rapidly evaporate near thermalization time.

Have very recently derived the equation for evolutions of condensate for elastic scattering small angle approximation.
Effect of Longitudinal Expansion

\[ \partial_t f - \frac{p_z}{t} \partial_{p_z} f = \frac{df}{dt} \bigg|_{p_z,t} = C[f] \quad \partial_t \epsilon + \frac{\epsilon + P_L}{t} = 0 \]

Assume approximate isotropy restored by scattering. Will check later that this is consistent.

\[ P_L = \delta \epsilon \quad 0 < \delta < 1/3 \]

\[ \varepsilon_g(t) \sim \varepsilon(t_0) \left( \frac{t_0}{t} \right)^{1+\delta} \quad \Lambda_s \sim Q_s \left( \frac{t_0}{t} \right)^{(4+\delta)/7}, \quad \Lambda \sim Q_s \left( \frac{t_0}{t} \right)^{(1+2\delta)/7} \]

\[ \left( \frac{t_{th}}{t_0} \right) \sim \left( \frac{1}{\alpha_s} \right)^{\frac{7}{3-\delta}} \]

For numerical results see works of Berges, Sexty, and Schlichting.
A large density of particles in a condensate can affect the previous considerations.

Condensate form in scalar field theory with no condensate initially

Overoccupied phase space evolve to both the UV and IR. In IR when it hits zero momentum it forms a condensate

In gauge theory, how to think about condensation
How to think about condensate formation:

Work in time like axial gauge:

\[ A^0 = 0 \]

\[ \delta A^i = A^i(t) - A^i(t = 0) \]

Under time independent residual gauge transformations:

\[ \delta A_i(x) \rightarrow U(x) \delta A_i(x) U^\dagger(x) \]

\[ tr(\delta A_i \delta A_i) \]

Is analogous to Higgs term in electroweak action, except it is an adjoint field that can condense
In Higgs model, adding a chemical potential for charge modifies the i-epsilon prescription and the static effective potential

\[ (M^2 - \mu^2) |\phi|^2 + \lambda |\phi|^4 \]

For positive mass squared for large enough density, a condensate is generated. In absence of vector fields, a Goldstone boson and a massive scalar (except when mass and chemical potential are equal) are generated. If there is a vector present, it eats the Goldstone boson and all modes become massive.

In abelian Higgs model: Can not really think about condensation in terms of Higgs field. Better to think about magnetic confinement, the Higgs phase, or a Coulomb phase where the magnetic flux is not confined. Vortices form in the Higgs phase, and are seen numerically.
Non-Abelian case:

Confinement of electric flux and magnetic flux are both order parameters

Screened magnetic and electric flux: High T QCD
Confined electric and screened magnetic: Low T QCD
Screened electric and and confined magnetic: Georgi-Glashow phase

Other possible phases: Gluons have spin and spin can get locked with color

We can try to understand this by numerical simulation, but also need to understand exactly what to look for.

There is a mystery here, tied in with these strong fluctuating fields, and it may ultimately teach us about confinement. It also has a life of its own.