Freeze-out Conditions in Heavy Ion Collisions: a lattice QCD based approach

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QCD phase diagram and conserved charge fluctuations in HIC

Search for QCD critical point in RHIC Beam Energy Scan

Higher order cumulants of conserved charge fluctuations:

Skewness: $S \sim \xi^{4.5}$

Kurtosis: $\kappa \sim \xi^{7}$

$\kappa < 0$ near the critical point

M. Stephanov

Measured by STAR, PHENIX
before looking for signatures of critical behavior, first need to establish basic non-critical features of conserved charge fluctuations are fluctuations described by equilibrium thermodynamics characterized by a unique set of $\left( T_f, \mu_B^f, \mu_Q^f, \mu_S^f \right)$?
QCD phase diagram and conserved charge fluctuations in HIC

are fluctuations described by equilibrium thermodynamics characterized by a unique set of \( (T^f, \mu^f_B, \mu^f_Q, \mu^f_S) \)?

are these parameters close to QCD transition line in the \( T - \mu_B \) plane?

are these parameters same as the chemical freeze-out parameters?
QCD transition line in the $T-\mu_B$ plane

$T_c(0) = 154(9)\text{ MeV}$

HotQCD

$T_c(\mu_B) = T_c(0) \left[ 1 - 0.0066(7)\mu_B^2 \right]$
strangeness deconfines at the chiral crossover from (lattice) QCD calculations
Chemical freeze-out and the transition line

LQCD: \[ T_c(\mu_B) = 154(9) \left[ 1 - 0.0066(7) \mu_B^2 \right] \text{ MeV} \]

BNL-Bielefeld

QCD critical point will be situated somewhere along this critical region

where do the conserved charge fluctuations freeze-out w.r.t the QCD transition line?

if they are not close enough then no signature of critical behavior in fluctuation related observables
Chemical freeze-out condition in HIC

\[ \sqrt{s} \]

Initial state

Pre-equilibrium

QGP, hydro. expansion

\[ T_c, \mu_c \]

Hadronization

Freeze-out

\[ T_f^c, \mu_B^f \]

Temperature

Time

\[ (T_f^{ch}, \mu_B^{ch}, \mu_Q^{ch}, \mu_S^{ch}) \]

by comparing hadron yields from HIC expt.

Statistical / thermal / HRG model

Chemical freeze-out

Hadron abundances unchanged

Thermalized, non-interacting hadrons & resonances
Chemical freeze-out condition in HIC

Chemical freeze-out is driven by inelastic scattering.

freeze-out conserved charge fluctuations is achieved via diffusion.

do conserved charge fluctuations freeze-out at the same chemical freeze-out point as hadron yields?
Fix freeze-out conditions \( \left( T_f, \mu_B^f, \mu_Q^f, \mu_S^f \right) \) by comparing experimental results for some lower order cumulants of conserved charge fluctuations with first-principle (lattice) QCD calculations.

If the fluctuations are described by equilibrium thermodynamics then all other higher order cumulants can be compared in a parameter-free manner with (L)QCD calculations to search for signatures of critical behavior in the experimentally measured observables.
fix using HIC initial conditions

strangeness neutrality

\[ \langle N_S \rangle = 0 \]

iso-spin asymmetry

\[ \langle N_Q \rangle = r \langle N_B \rangle \]

assume: homogeneous system

\[ \langle n_S \rangle = 0 \]

\[ \langle n_Q \rangle = r \langle n_B \rangle \]

\[ N_X = N_X - N_{\bar{X}}, \quad n_X = N_X/V \]

B: baryon, Q: charge, S: strangeness

p: proton, n: neutron

Au-Au & Pb-Pb: \( r \approx 0.4 \)
\[ \langle n_S \rangle = 0 \quad \langle n_Q \rangle = r \langle n_B \rangle \]

expand these two relations in powers of \( \mu_B, \mu_Q, \mu_S \) around \( \mu_B = \mu_Q = \mu_S = 0 \)

\[ \mu_Q(T, \mu_B) = q_1(T) \mu_B + q_3(T) \mu_B^3 + \cdots \]

\[ \mu_S(T, \mu_B) = s_1(T) \mu_B + s_3(T) \mu_B^3 + \cdots \]
Strangeness and electric charge chemical potentials

\[ \langle n_S \rangle = 0 \quad \langle n_Q \rangle = r \langle n_B \rangle \]

expand these two relations in powers of \( \mu_B, \mu_Q, \mu_S \) around \( \mu_B = \mu_Q = \mu_S = 0 \)

\[
\mu_Q(T, \mu_B) = q_1(T) \mu_B + q_3(T) \mu_B^3 + \cdots \\
\mu_S(T, \mu_B) = s_1(T) \mu_B + s_3(T) \mu_B^3 + \cdots
\]

**NLO:** cumbersome but straightforward

**LO:** for example

\[
\langle n_s \rangle = (\chi_{11}^{BS} + q_1 \chi_{11}^{QS} + s_1 \chi_2^S) \mu_B + \cdots = 0 \\
(\chi_{11}^{BQ} + q_1 \chi_2^Q + s_1 \chi_{11}^{QS}) \mu_B + \cdots = r (\chi_2^B + q_1 \chi_{11}^{BQ} + s_1 \chi_{11}^{BS}) \mu_B + \cdots
\]

2 equations, solve for 2 unknowns: \( q_1, s_1 \)
Strangeness and electric charge chemical potentials

\[ \mu_Q(T, \mu_B) = q_1(T) \mu_B + q_3(T) \mu_B^3 \]

\[ \mu_S(T, \mu_B) = s_1(T) \mu_B + s_3(T) \mu_B^3 \]

**LO:** \( N_\tau = 6, 8, 12 \) continuum extrapolated

**NLO:** corrections < 10\% for \( T > 140 \text{ MeV}, \frac{\mu_B}{T} \lesssim 1.3 \)

\( N_\tau = 6, 8, \) small cut-off dependence, *continuum estimate*
Strangeness and electric charge chemical potentials

\[-\mu_Q(T, \mu_B)/\mu_B\]

NLO expansion is well controlled for 

\[T = 150 - 170 \text{ MeV}, \ \mu_B \approx 200 \text{ MeV}\]

covers RHIC energies 

\[\sqrt{s_{NN}} \gtrsim 20 \text{ GeV}\]

\[\mu_S(T, \mu_B)/\mu_B\]

\(~10\% \text{ deviations from HRG}\)

BNL-BI: PRL 109, 192302 (2012)
Strangeness and electric charge chemical potentials

\[ \frac{\mu_s}{\mu_B} = 0.2 - 0.3 \]

weak dependence on \( \mu_B \)

how does it compare with the chemical freeze-out parameters obtained from thermal model fits to hadron yields?
Strangeness and electric charge chemical potentials

\[ \mu_S / \mu_B = 0.2 - 0.3 \]

weak dependence on \( \mu_B \)

reflects strangeness neutrality
Temperature and baryon chemical potential

all observables can now be obtained as function of two independent parameters $T, \mu_B$

comparison of 2 expt. measured ratios of cumulants of conserved charge fluctuations with LQCD calculations fixes 2 freeze-out parameters $T^f, \mu_B^f$

proton fluctuations (expt.) $\approx$ baryon fluctuations (LQCD)

Asakawa-Kitazawa; Bzdak-Koch-Skokov

safe to work with net electric charge fluctuations measured both in expt. and LQCD

ratio of cumulants: cancels the unknown volume of the fireball
Temperature and baryon chemical potential

\[
\frac{M_Q(\sqrt{s})}{\sigma_Q(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_Q(T, \mu_B)}{\chi_Q(T, \mu_B)} = R_{12}^Q(T)\mu_B + R_{12}^Q(T)\mu_B + \cdots = R_{12}^Q(T, \mu_B)
\]

\[
\frac{S_Q(\sqrt{s})\sigma^3_Q(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3(T, \mu_B)}{\chi_1(T, \mu_B)} = R_{31}^0(T) + R_{31}^2(T)\mu_B^2 + \cdots = R_{31}^Q(T, \mu_B)
\]

**baryometer**, fixes \(\mu_B^f\)

**thermometer**, fixes \(T_f\)

**HIC**
- mean: \(M_Q\)
- variance: \(\sigma_Q^2\)
- skewness: \(S_Q\)
- \(\delta N_Q = N_Q - \langle N_Q \rangle\)

**STAR, PHENIX**

**LQCD**

generalized charge susceptibilities:

\[
\chi_n(T, \bar{\mu}) = \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \bar{\mu})}{\partial (\mu_Q/T)^n}
\]
The baryometer

\[ R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \]

\[ R_{12}^Q(T, \mu_B) = R_{12}^{Q,1}(T)\mu_B + R_{12}^{Q,3}(T)\mu_B^3 \]

**LO:** \( N_\tau = 6, 8, 12 \) continuum extrapolated

**NLO:** corrections < 10\% for \( T > 140 \) MeV, \( \mu_B/T \leq 1.3 \)

\( N_\tau = 6, 8 \), small cut-off dependence, *continuum estimate*
Importance of $\mu_Q \neq 0, \mu_S \neq 0$

$R_{12}^{Q,1} = \frac{\chi_{11}^{BQ}}{\chi_2^Q} + \frac{\mu_Q}{\mu_B} + \frac{\mu_S}{\mu_B} \frac{\chi_{11}^{QS}}{\chi_2^Q}$

contributes equally in the relevant $T$ range

2+1 flavor QCD essential
The thermometer

\[ R_{31}^Q = S_Q \sigma_Q^3 / M_Q \]

large deviation from HRG for \( T > 155 \) MeV

provide stringent constraint on \( T \)

\[ R_{31}^Q(T, \mu_B) = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T) \mu_B^2 \]

NLO corrections: \( \lesssim 10\% , \mu_B / T \lesssim 1.3 \)
\[ R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \]

\[ R_{31}^Q = S_Q \sigma_Q^3 / M_Q \]

\[ \mu_B/T = 1 \]
\[ \mu_B/T = 0 \]
\[ N_T = 6 \]
\[ N_T = 8 \]

<table>
<thead>
<tr>
<th>$S_Q \sigma_Q^3 / M_Q$</th>
<th>$T_f$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 2$</td>
<td>$\leq 155$</td>
</tr>
<tr>
<td>$\sim 1.5$</td>
<td>$\sim 160$</td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>$\geq 170$</td>
</tr>
</tbody>
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Systematic: estimates using HRG

**series truncation:** \( \sim 5\% \)

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**continuum limit:** \( \sim 5\% \)
STAR data on net proton fluctuations

Star preliminary
X. Luo, Quark Matter 2012
STAR and PHENIX data on electric charge fluctuations

**STAR Preliminary**
L. Kumar, Quark Matter 2012

**PHENIX Preliminary**
J. Mitchell, Quark Matter 2012
$R_{31}^Q = S_Q \sigma_Q^3 / M_Q$

$T_f$ is in reasonably good agreement with the chemical freeze-out temperature $T_{f, ch}^f \approx 160$ MeV

but at present large experimental errors for a more precise determination
variation of $T_{f,\mathrm{ch}}$ is $< 5$ MeV for $\sqrt{s_{\mathrm{NN}}}>19$ GeV

as a first start, use the average value over $\sqrt{s_{\mathrm{NN}}}=19.6-200$ GeV

$R_{31}^Q = S_Q \sigma_Q^3/M_Q$

$R_{31}^Q = 1.56(16)$

STAR preliminary
Quak Matter 2012

for orientation:

$T_c(\mu_B=0)=154(9)$ MeV

$T_{f,\mathrm{ch}}=160(5)$ MeV

$T_f=158(7)$ MeV
$R_{12}^Q = M_Q / \sigma_Q^2$

$\mu_B^f / T^f$ agree reasonably with $\mu_B^{f,\text{ch}} / T^{f,\text{ch}}$
net-proton fluctuations == net-baryon fluctuations

\[ R_{12}^{B} = \frac{M_B}{\sigma_B^2} \text{ vs. } R_{12}^{p} = \frac{M_p}{\sigma_p^2} \]

Moreover, \( \mu_B^f / T^f \) agree reasonably with \( \mu_{B,\text{ch}}^f / T_{\text{ch}}^f \).
Thermodynamic consistency

if the fluctuations are described by equilibrium thermodynamics

\[ R_{12}^Q / R_{12}^B = \frac{R_{12}^Q}{R_{12}^B} \]

must contain identical information regarding \( T \) and \( \mu_B \)
Thermodynamic consistency

if the fluctuations are described by equilibrium thermodynamics

However ...

currently STAR preliminary @ $\sqrt{s_{\text{NN}}}=200$ GeV: $R_{12}^Q/R_{12}^B \approx 0.06$

a problem !!
Thermodynamic consistency

in other words ...

give inconsistent values for $\mu_B^f$
a problem !!
are conserved charge fluctuations in HIC consistently described by equilibrium thermodynamics characterized by a unique set of freeze-out parameters ??

can be tested and the freeze-out parameters can be determined by comparing HIC experimental results with first-principle (lattice) QCD calculations via controlled NLO Taylor expansion up to

\[ \mu_b \approx 200 \text{ MeV}, \sqrt{s_{NN}} \approx 19.6 \text{ GeV} \]

general agreement with chemical freeze-out parameters obtained from thermal model fits to hadron yields, but some problem remains to be addressed once such a procedure is fully carried out, then (lattice) QCD calculations can be compared with other experimentally measured higher cumulants to search for signatures of critical behavior