It must be something!

- Fully Artistic!
- Art’s playground for many years
- Fun for everyone!
Anisotropic Flow

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Wayne State University, Detroit, Michigan

Disclaimer:

- Not a real review – this is an introduction to Anisotropic Flow: terminology, physics, analysis techniques, achievements and problems, current status of the art.

- Many of the most recent results will be discussed in the original talks at this School.

- Apologies for not mentioning many good papers on the subject. This talk is based mostly on works where I participated and know the best.
Outline

Part I. Physics

1. Introduction. Definitions.

2. Directed flow
   - Physics of the “wiggle”
   - Blast wave parameterization
   - Coalescence I. Directed flow of light nuclei

3. Elliptic flow
   - General properties. “Early times”
   - Low Density and “Hydro” Limits
   - Phase transition
   - Blast wave. “Mass splitting”
   - Coalescence II. Constituent quarks. Resonances.
   - Elliptic flow at high pt’s.

4. Anisotropies and asymmetries
   - Femtoscopy of anisotropic source
   - High pt, 2-particle correlations
   - Global polarization, Parity violation...

5. Can we measure it?
   - correlations induced by flow

Part II. Methods and Results

1. Non-flow estimates
   - From the “resolution plot”
   - Azimuthal correlations in pp and AA

2. Multiparticle correlations
   - 4-particle cumulants. Methods.
   - Non-flow and flow fluctuations
   - Mixed harmonics. 3-particle correlations.
   - Distributions in q-vector
   - Detector effects

3. Main results
   - Reaching the hydro limit
   - Mass splitting
   - Constituent quark scaling
   - Elliptic flow at high pt
   - $v_1$ and higher harmonics
   - other

4. Conclusion
Anisotropic flow ≡ correlations with respect to the reaction plane

Term “flow” does not mean necessarily “hydro” flow – used only to emphasize the collective behavior ↔ multiparticle azimuthal correlation. Newer trend: “event anisotropy”

No symmetry between “x” and “-x”, except midrapidity
Symmetry between “y” and and “-y” (Otherwise – parity violation)

No symmetry in the configuration space → anisotropy in the momentum space: \( \frac{dN}{d\phi} \neq \text{const} \)
Term “flow” does not mean necessarily “hydro” flow – used only to emphasize the collective behavior ↔ multiparticle azimuthal correlation.

Newer trend: “event anisotropy”

Anisotropic flow ≡ correlations with respect to the reaction plane

Note large orbital angular momentum in the system!

No symmetry between “x” and “-x”, except midrapidity
Symmetry between “y” and and “-y” (Otherwise - parity violation)

No symmetry in the configuration space → anisotropy in the momentum space: \( \frac{dN}{d\phi} \neq \text{const} \)
How to characterize anisotropic flow?

\[ \langle p_x \rangle, \text{ 3d sphericity, 2d sphericity} \]

\[ S_{ij} = \langle p_i p_j \rangle \]

\[ i, j = 1, 2, 3 \text{ or } 1, 2 \]

**Fourier decomposition** of single particle (semi) inclusive spectra:

\[
\frac{d^3 N}{dp_t \, dy \, d\phi} = \frac{d^2 N}{dp_t \, dy} \frac{1}{2\pi} (1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \ldots)
\]

Advantages:
- Describes different kind of anisotropies in a common way
- Possibility to fully correct the results \( \rightarrow \) compare directly with theory and other experiments
Definitions, continued

\[
\frac{d^3N}{dp_t \, dy \, d\phi} = \frac{d^2N}{dp_t \, dy} \frac{1}{2\pi} (1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \ldots)
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\[v_n = \langle \cos(n(\phi - \Psi_r)) \rangle\]

Terminology:
- harmonics, not multipoles (dipole, etc. required to describe 2d or 3d distributions)

“... \(v_2\) in pp collisions is almost 100% ...”
“... event anisotropy at high \(p_t\), elliptic flow at low \(p_t\)...”

Anisotropic flow \(\equiv\) correlations with respect to the reaction plane
Definitions, continued

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Directed flow

Elliptic flow

Anisotropic flow \equiv \text{correlations with respect to the reaction plane}

The situation with definitions is more complicated with two particle spectra measured with respect to the reaction plane. One way: parameterize, consider parameter dependence on pair orientation relative to the reaction plane.

\[
\frac{dn_{\text{pair}}}{d\Delta \phi} = F(\Delta \phi; \Psi_{RP})
\]
Why are we interested in anisotropic flow?

Short answer: to learn more about system properties, evolution dynamics, hadronization - Anisotropic flow - as a measure of interactions in the system.


PRD 46 (1992) 229
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Collide Au+Au, is it enough to create QGP?

- Is the system dense enough? What are the Quark & Gluon densities?
- Does the system equilibrate?
- What is the initial Temperature?

To answer these questions we need to study the system early in the collision
- hard (rare) probes: J/Psi, jets, dileptons,...
- anisotropic flow!

Before discussing how the anisotropic flow let us set some time scales (in the center of mass frame):

Passing time - the time two nuclei pass each other ~ $R_A/\gamma$

Initial transverse size of the system ~ $R_A$

Freeze-out time (could be up to a several $R_A$)

Directed flow is predefined at passing time, as at this moment the initial geometry of the system being set.

Elliptic flow is mostly defined at the time scale of the initial size of the system for two reasons:
- This is the time required for a fast particle to traverse the system
- The time after which the spatial anisotropy diminish
Qualitative features of anisotropic flow.

Let us try to “catch” the possible physics:

Can we describe flow assuming some properties of the collective motion?

Directed flow: does it look like particle emission from
a) Moving source?
b) Screened source (shadowing)?

Elliptic flow:
- Surface emission?
- Anisotropically expanding source?
- Rescattering?

Among others, a few pictures will be discussed in more detail:

a) Blast wave “parameterization”
b) Low Density Limit
c) Coalescence
Directed flow “wiggle” in cascade models

Snellings, Poskanzer, S.V., nucl-ex/9904003

Snellings, Sorge, S.V., F. Wang, Nu Xu, PRL 84 (2000) 2803

FIG. 2. RQMD calculations of $v_1$ (filled circles) and $s_1$ (open circles) for nucleons (left panel) and pions (right panel).

The wiggle is pronounced only at high energies.
Directed flow “wiggle” in cascade models

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<th>Rapidity</th>
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</tr>
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Baryon stopping

Radial flow

\( \langle x p_x \rangle > 0 \)

“wiggle”

Does the picture contradict FOPI results on different isotope collisions?
Hydro: “antiflow”, “third flow component”


Brachmann, Soff, Dumitru, Stocker, Maruhn, Greiner Bravina, Rischke, PRC 61 (2000) 024909

Figure 2: Au+Au collision at $\sqrt{s_{NN}} = 100$ GeV/nucleon, ($b = 0.5 \cdot 2 R_{Au}$), $E= T_{500}$ is presented in the reaction plane as a function of $z$ and $x$ for $t_b = 5$ fm/c. Subplot A) $A = 0.065$, subplot B) $A = 0.08$. The QGP volume has a shape of a tilted disk and may produce a third flow component.
Third flow component as the QGP signal

L.P. Csernai, D. Rohrich
PRL 458 (1999) 454

\[ \langle p_x \rangle \approx v_1 \langle p_t \rangle \]

The "wiggle" is present only for the QGP EoS.

This calculations have been done at 11 AGeV. Would the results change for RHIC?

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Fig. 1. Upper part: Definition of the measure softening, $S$, describing the deviation of $P_x(y)$ or $v_1(y)$ from the straight line behavior, $ay$, around CM. $S$ is defined as $|ay - P_x(y)|/|ay|$. The lower figure shows a typical example for fluid dynamical calculations with Hadronic and QGP EoS [3]. QGP leads to strong softening, $\sim 100\%$. 
Interplay of radial and anisotropic (directed) flow

\[ v_1(p_t) \approx \frac{p_t \beta_x}{2T} \left( 1 - \frac{m\beta_r}{p_t} \frac{I_1(\beta_r p_t / T)}{I_0(\beta_r p_t / T)} \right) \]

\[ \beta_x = 0.1 \quad T = 120 \text{ MeV} \quad (\beta_{\text{thermal}} \approx 0.5) \]

Interplay of three velocities:
1) Thermal velocity
2) Radial expansion mean velocity
3) Anisotropic (modulation in radial expansion)

The effect is larger for larger mass
Interplay of radial and anisotropic (directed) flow

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High pt particles are produced mostly in the red region; low pt particles - in the blue region

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Note! Similar formalism can be achieved with totally different interpretation (not requiring thermalization), where the role of Temperature plays mean pt change due to scattering
Radial flow velocity - mean radial component of particle velocity
Anisotropic flow velocity - the modulation in the above
Fitting the real data


Directed flow of protons in Au+Au collisions at $E_{\text{lab}}=11.4$ GeV, $2.6 < y < 2.8$

$v_1(p_t) \approx \frac{p_t \beta_x}{2T} \left(1 - \frac{m\beta_r}{p_t} \frac{I_1(\beta_r p_t / T)}{I_0(\beta_r p_t / T)}\right)$

$T = 110$ MeV, $y - y^* = 0.5$
Coalescence I. E877 light nuclei flow.

What is needed for the equation above to work?

a) “Rare” process
b) \( B=\text{const} \) only if the configuration space density does not depend on the orientation wrt RP

Note! The coalescence picture itself can have much larger region of applicability than the equation above. We/I just do not know how to describe coalescence in the case of “not rare processes”.

E877 conclusion:
Configuration space density increases in the direction of flow.
Coalescence I. E877 light nuclei flow.

\[
\frac{d^3 N_d}{d^3 p} (p) \propto B \left( \frac{d^3 N_p}{d^3 p} \left( \frac{p}{2} \right) \right)^2 \rightarrow
\]

\[
v_{1,d} (p_t) \approx 2v_{1,p} \left( \frac{p_t}{2} \right) / \left( 1 + 2v_{1,p}^2 \right)
\]

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Configuration space density increases in the direction of flow.

Note \( v_1 \) values > 0.5 → there must be other non-zero harmonics!
Resonances


School of Collective Dynamics in High Energy Collisions, LBNL, May 19-27, 2005

Centrality 1

\( \pi^- \)

\( \pi^+ \)

Centrality 2

\( p_t \) (GeV/c)

Coulomb Interaction

Static Coulomb source shifted in \( x \) direction by \( \langle r \rangle \):

\[ v_1 = \langle r \rangle / a \approx \frac{\langle r \rangle Z}{(200 \text{ fm})} \quad (k \to 0) \]
Resonances

**Δ (and Λ) DECAYS.**

\[ \beta_x \approx \beta_x^{(\Delta)} \]

\[ \beta_\pi \approx 0.85 \ (= \beta_r^{"\prime\prime}) \]

\[ \beta_{\text{thermal}} \approx \beta_{\text{thermal}}^{(\Delta)} \]

**Note:** one of the examples of "non-thermal" interpretation of blast wave formulae

Elliptic flow:

Elliptic flow must vanish if initially the system was created symmetric. Then, at small eccentricities, $v_2 \sim \varepsilon$.

Other similar/same quantities:
- Ollitrault: $\alpha_s$
- Heiselberg: $\delta$
- Sorge: $A_2$
- Shuryak: $s_2$

Note:
-- it is not at all trivial what should be used (if any) for higher harmonics (no simple form)
-- $s_2$ parameter in the Blast Wave fit (below) to $v_2(p_t)$, in general, is a different parameter
-- do not confuse initial and final state anisotropy
Elliptic flow. General properties.

Elliptic flow must vanish if initially the system was created symmetric. Then, at small eccentricities, \( v_2 \sim \varepsilon \).

\[
\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}
\]

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- do not confuse initial and final state anisotropy

"e" -- initialization of energy density; "s" - initialization of entropy density

**FIG. 4.** Initial spatial anisotropy as a function of impact parameter, for the different initializations.

Figure 4 shows the initial spatial anisotropy

\[
\varepsilon_x(b) \equiv \delta(b) = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}
\]

as a function of impact parameter, evaluated with the initial transverse energy density as weight function.
The sensitivity is due to two reasons:

a) Decrease in spatial anisotropy

b) Decrease in spatial particle density.

\[ \varepsilon = \frac{\left\langle y^2 - x^2 \right\rangle}{\left\langle y^2 + x^2 \right\rangle} = \frac{\left\langle \left( y_0 + vt \sin \phi \right)^2 - \left( x_0 - vt \cos \phi \right)^2 \right\rangle}{\left\langle \left( y_0 + vt \sin \phi \right)^2 + \left( x_0 - vt \cos \phi \right)^2 \right\rangle} = \frac{\left\langle y_0^2 - x_0^2 \right\rangle}{\left\langle y_0^2 + x_0^2 + (vt)^2 \right\rangle} \]

\[(16-9)/(16+9+x^2)\]
When the elliptic flow is formed

XZ-plane - the reaction plane

$\mathcal{E} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$

$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos(2\varphi) \rangle$

Sensitive to the physics of constituent interactions (needed to convert space to momentum anisotropy) at early times (free-streaming kills the initial space anisotropy)

Zhang, Gyulassy, Ko, PL B455 (1999) 45

The characteristic time scale of 2-4 fm is similar in any model: parton cascade, hydro, etc.
Low density limit

\[ \frac{dN}{d\varphi} = \left( \frac{dN}{d\varphi} \right)_0 + \Delta \left( \frac{dN}{d\varphi} \right) \quad n(v) = n_0(v) + \Delta n(v) \]

Change in the particle flux is proportional to the probability for the particle to interact.

\[ \Delta n(v) \propto \int dr_0 \rho(\mathbf{r}_0) \int dt \rho(\mathbf{r}_0 + vt, t) \quad \rho(\mathbf{r}_0 + vt, t) \propto \int dv' \rho(\mathbf{r}_0 - v't) \]

Integrations over: a) particle emission point
b) Over the trajectory of the particle (time) with
weight proportional to the density of other particles
--“scattering centers”

\[ \Delta n(v) \propto \int dr_0 \rho_0(\mathbf{r}_0) \int dt \int dv' \rho_0(\mathbf{r}_0 - (v - v')t) \]

\[ \rho_0 \propto \exp \left( -\frac{x^2}{2\langle x^2 \rangle} - \frac{y^2}{2\langle y^2 \rangle} \right) \]

\[ v_2^i = \frac{\varepsilon}{16\pi R_x R_y} \sum_j \langle v_{ij} \sigma_{ij}^\text{transport} \rangle \frac{dN_j}{dy} \frac{v_{i\perp}^2}{v_{i\perp}^2 + \langle v_{j\perp}^2 \rangle} \]

\[ v_2 \propto \frac{1}{S} \frac{dN}{dy} \quad S = \pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle} \]
Low density limit

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\]

\[
v_2 \propto \varepsilon \frac{1}{S} \frac{dN}{dy}
\]

\[
S = \pi \sqrt{\left\langle x^2 \right\rangle \left\langle y^2 \right\rangle}
\]

Note:
(x-vt) changes very little over the entire history
First hydro calculations

In hydro, where mean free path is by assumption much less than the size of the system, there is no other parameters than the system size (may enter time scales, see below). Then elliptic flow must follow closely the initial eccentricity.
Centrality dependence

\[ \frac{v_2}{\varepsilon} \]

\[ \frac{1}{S} \frac{dN}{dy} \]

LDL parameters: mean free path, \( \varepsilon \)
Hydro: only \( \varepsilon \), may enter time scale

"HYDRO limit"

Ollitrault: \( \frac{v_2}{\varepsilon} \approx \frac{v_2^{(p^2)}}{2\varepsilon} \approx 0.27 \pm 0.35 \)

Heinz et al.: \( (v_2/\varepsilon)_{HYDRO} \approx 0.21 \pm 0.23 \)

S.V. & A. Poskanzer, PLB 474 (2000) 27

Points – RQMD, dashed curve - \( \varepsilon(b) \)
More on "hydro limits"

Suppressed scale!

\[ v_2 = 0.04 \frac{\langle v_\perp \rangle}{\langle dN / dy \rangle / 300} \]

Minimum in \( v_2 / \varepsilon \) due to softening of the EoS at phase transition

Uncertainties:
Hydro limits: slightly depend on initial conditions
Data: no systematic errors, shaded area - uncertainty in centrality determinations.
Curves: “hand made”
**\( \nu_2/\varepsilon \) and phase transitions**

What to expect for different nuclei, Cu+Cu?
First guess would be:

\[
\nu_2 \propto \frac{1}{\eta} \frac{dN}{dy} \frac{1}{R_A}
\]

**Uncertainties:**
- Hydro limits: slightly depend on initial conditions
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S.V. & A. Poskanzer, PLB 474 (2000) 27
**v_2/ε and phase transitions**

S.V. & A. Poskanzer, PLB 474 (2000) 27

What to expect for different nuclei, Cu+Cu?
First guess would be:

\[ v_2 \propto \frac{1}{\varepsilon} \frac{dN}{S \ dy} \frac{1}{R_A} \]

... but there is no such factor in the LDL:

\[ v_2' = \frac{\varepsilon}{16\pi R_x R_y} \sum_j \left\langle v_{ij} \sigma_{transport} \right\rangle \frac{dN_j}{dy} \frac{v_{i\perp}^2}{v_{i\perp}^2 + \left\langle v_{j\perp}^2 \right\rangle} \]

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Centrality dependence. Hydro + RQMD.

LH8 ≡ latent heat = 0.8 GeV/fm^3
Pt slope parameters are about 20% larger in hydro compared to data

- $v_2$ increases with dN/dy
- Centrality dependence - close to data
$v_2(p_t)$ - Houvinen, Kolb, Heinz, Ruuskanen, S.V., PLB 503 (2001) 58

$v_2(p_t) = \frac{\int_{0}^{2\pi} d\phi_s \cos(2\phi_s) I_2(\alpha_t) K_1(\beta_t)}{\int_{0}^{2\pi} d\phi_s I_0(\alpha_t) K_1(\beta_t)}$

Parameters:
- $T$ - temperature
- $\rho_0$ - radial expansion rapidity
- $\rho_2$ - amplitude of azimuthal variation in expansion rapidity

$\alpha_t = (p_t / T) \sinh(\rho)$; $\beta_t = (m_t / T) \cosh(\rho)$;

$\rho = \rho_0 + \rho_a \cos(2\phi_s)$

$\oplus$ Elementary source density - $\propto 1 + 2s_2 \cos(2\phi_s)$

Note:
- a) The possibility different interpretation of the parameters (other than “hydro-like”)
- b) The possibility of different “realization” of the parameter $s_2$. There is no strict correspondence between this parameter and the shape of the source at freeze-out.
How much $s_2$ matters?

Parameters:
- $T$ - temperature
- $\rho_0$ - radial expansion rapidity
- $\rho_2$ - amplitude of azimuthal variation in expansion rapidity

Elementary source density -
$$\propto 1 + 2s_2 \cos(2\varphi_s)$$

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Elementary source density -

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- model fits data well
- shape ($s_2$ parameter) agrees with the interferometry measurements (see below) under assumption that flow velocity field is normal to the surface
$v_2(p_T)$. "Mass splitting". Dependence on the EoS.

"Mass splitting" is stronger for the QGP EoS.

But qualitatively such a mass dependence will be present in any model, for example, in the constituent quark coalescence picture (discussed below)

(heavier particle $\leftrightarrow$ larger difference in constituent quark momenta)
Constituent quark model + coalescence

coalescence

Only in the intermediate region (rare processes) coalescence can be described by:

\[ \frac{d^3 n_M}{d^3 p_M} \propto \left[ \frac{d^3 n_q}{d^3 p_q} \left( p_q \approx p_M / 2 \right) \right]^2 \rightarrow v_{2,M}(p_t) \approx 2 v_{2,q}(p_t / 2) \]

\[ v_{2,B}(p_t) \approx 3 v_{2,q}(p_t / 3) \]

In the low \( p_t \) region the density is large and most quarks coalesce:

\[ N_{\text{hadron}} \sim N_{\text{quark}} = e^{-B p_t^2} < e^{-B (p_t / 2)^2} \]

In the high \( p_t \) region fragmentation eventually wins:

\[ p_t^{-n} \gtrsim \left[ (p_t / 2)^{-n} \right]^2 \]

Taking into account that in coalescence

\[ p_{t,\text{quark}} \approx p_{t,\text{meson}} / 2 \]

and in fragmentation

\[ p_{t,\text{quark}} \approx p_{t,\text{meson}} / z \]

there could be a region in quark \( p_t \) where only few quarks coalesce, but give hadrons in the hadron \( p_t \) region where most hadrons are produced via coalescence.

---

School of Collective Dynamics in High Energy Collisions, LBNL, May 19-27, 2005  S.A. Voloshin
Constituent quark model + coalescence

Coalescence

Low $p_t$ quarks

Only in the intermediate region (rare processes) coalescence can be described by:

$$\frac{d^3n_{M}}{d^3p_{M}} \propto \left[ \frac{d^3n_q}{d^3p_q} \left( p_q \approx \frac{p_M}{2} \right) \right]^2 \Rightarrow v_{2,M}(p_t) \approx 2v_{2,q}(p_t / 2)$$

$$v_{2,B}(p_t) \approx 3v_{2,q}(p_t / 3)$$

In the low $p_t$ region the density is large and most quarks coalesce:

$$N_{hadron} \sim N_{quark} \quad e^{-Bp_t^2} \ll \left( e^{-B(p_t/2)^2} \right)^2$$

In the high $p_t$ region fragmentation eventually wins:

$$p_t^{-n} \gg [(p_t / 2)^-n]^2$$

Taking into account that in coalescence $p_{t,quark} \approx p_{t,meson} / 2$

and in fragmentation $p_{t,quark} \approx p_{t,meson} / z$

there could be a region in quark $p_t$ where only few quarks coalesce, but give hadrons in the hadron $p_t$ region where most hadrons are produced via coalescence.

Side-notes:

a) more particles produced via coalescence rather than parton fragmentation $\Rightarrow$ larger mean $p_t$...

b) $\Rightarrow$ higher baryon/meson ratio

c) $\Rightarrow$ lower multiplicity per "participant"

- \(\Rightarrow\) D. Molnar, QM2004
- \(\Rightarrow\) Bass, Fries, Müller, Nonaka; Levai, Ko; …
- \(\Rightarrow\) Eremin, S.V.

S.V., QM2002
D. Molnar, S.V., PRL 2003
Elliptic flow due to jet quenching

Gyulassy, Vitev & Wang, PRL 86 (2001) 2537

R. Snellings, A. Poskanzer, S.V., nucl-ex/9904003

Easier “escape” in the x direction ➔ “in-plane” particle emission
Hard shell      ==  box density profile (+) extreme quenching
Hard sphere   ==     -"-  (+) realistic quenching
Woods-Saxon ==  WS density profile  (+) realistic quenching

Probability to “escape”:  \( \propto \exp(-\kappa L) \)

\( v_2^{[4]}(3 \text{ GeV}/c < p_t < 6 \text{ GeV}/c) \)

STAR, PRL 93 (2004) 252301
New possibilities

Anisotropies and asymmetries

1. HBT with respect to the reaction plane
2. Non-identical particle correlations with respect to the reaction plane
3. High pt 2-particle correlation wrt reaction plane
4. Global polarization in AA
5. Parity violation
Let us measure the geometry of anisotropic source!


Further technique developments: U. Wiedemann, U. Heinz, M. Liza

First attempt to measure: D. Miskowiec, E877, QM ’95

First and subsequent real measurements: M. Lisa et al., E895, STAR

---

**TABLE II.** Matrix of source radii (in fm$^2$) found using the HBT correlation function measured from the different directions with respect to flow in the center of mass system of the colliding nuclei. The input parameters are taken from Table I.

<table>
<thead>
<tr>
<th></th>
<th>$R_x^2$</th>
<th>$R_y^2$</th>
<th>$R_z^2$</th>
<th>$R_{xy}^2$</th>
<th>$R_{xz}^2$</th>
<th>$R_{yz}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x+$</td>
<td>16.9</td>
<td>19.1</td>
<td>12.6</td>
<td>0.8</td>
<td>-1.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>$x-$</td>
<td>25.3</td>
<td>16.6</td>
<td>11.8</td>
<td>-1.1</td>
<td>3.3</td>
<td>1.2</td>
</tr>
<tr>
<td>$y+$</td>
<td>17.6</td>
<td>24.0</td>
<td>12.9</td>
<td>-1.9</td>
<td>2.7</td>
<td>0.6</td>
</tr>
<tr>
<td>$y-$</td>
<td>15.8</td>
<td>24.8</td>
<td>14.7</td>
<td>2.8</td>
<td>1.2</td>
<td>-0.2</td>
</tr>
</tbody>
</table>
Hanbury Brown – Twiss interferometry of an anisotropic source

Let us measure the geometry of anisotropic source!

Further technique developments: U. Wiedemann, U. Heinz, M. Liza
First attempt to measure: D. Miskowiec, E877, QM '95
First and subsequent real measurements: M. Lisa et al., E895, STAR

Can we see the evolution?

<table>
<thead>
<tr>
<th></th>
<th>$R_x^2$</th>
<th>$R_y^2$</th>
<th>$R_z^2$</th>
<th>$R_{xy}^2$</th>
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<td>-0.2</td>
</tr>
</tbody>
</table>
Blast wave model:

$T=100$ MeV, $\langle \rho_0 \rangle = 0.6$

$R = 11.7$ fm, $\Delta \tau = 2.2$ fm/c

$\langle \rho_a \rangle = 0.037$, $\langle s_2 \rangle = 0.037$

FIG. 2 (color online). Squared HBT radii relative to the reaction plane angle for four $k_T$ (GeV/c) bins, 20%–30% centrality events. The solid lines show allowed [24] fits to the individual oscillations.
Blast wave model:

\[ T = 100 \text{ MeV}, \langle \rho_0 \rangle = 0.6 \]
\[ R = 11.7 \text{ fm}, \Delta \tau = 2.2 \text{ fm/c} \]
\[ \langle \rho_a \rangle = 0.037, \langle s_2 \rangle = 0.037 \]

Same parameters fit \( R(\phi) \) and \( v_2(p_T, m) \)

FIG. 30. Final source eccentricity (\( \varepsilon_{\text{final}} \)) as calculated from the Fourier coefficients \( \frac{2R_{s,2}^2}{R_{s,0}^2} \) and from the final in-plane and out-of-plane radii \( \left( \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2} \right) \) vs. initial eccentricity (\( \varepsilon_{\text{initial}} \)). The most peripheral collisions correspond to the largest eccentricity. The line indicates \( \varepsilon_{\text{final}} = \varepsilon_{\text{initial}} \). Systematic errors of 30\%, based on sensitivity to model parameters [58], are assigned to \( \varepsilon_{\text{final}} \) extracted from the Fourier coefficients.
Elliptic anisotropy of pion source for different pion $p_t$ regions. Pion rapidity $-0.5 < y < 0.5$.

$$\frac{1}{R_s^2} \approx 1 + \frac{1}{L_x^2 \left[ v_{th} / \left( dV_{side} / dr_{side} \right) \right]^2}$$

$v_{th} = \sqrt{T / m}$ - thermal velocity

$dV / dx$ - expansion velocity gradient

$t$ - expansion time

$L$ - source size at freeze-out

$R_{side}$ as a function of pion emission angle relative to the reaction plane for three $p_t$ regions. Impact parameter cut $3 < b < 6$.

The 180 degree (open) points are the reflection of the -180 degree points.

In this picture at high energies / high $p_t$, the relative difference between out-of-plane size and in-plane size only increases.
Note “out-of-phase” $R_{\text{side}}$ modulations for $k=0$ case.

Should we try very low $k_T$ at RHIC?

IPES initial conditions, U. Heinz, P. Kolb PL B542 (2002) 216
Do different particles freeze-out at the same place?

RQMD v2.3, PbPb, 158 GeV/nucleon

6 < b < 12 - centrality

cos#phi > 0.7 - orientation wrt RP

0.6 < Pt/m_t < 0.8

\[ X \]

\[ Y \]

rapidity

rapidity
Non-identical 2-particle correlations

\[ k_i^* > 0 \text{ and } k_i^* < 0, \text{ as } R_i^{(+)} \text{ and } R_i^{(-)} \]

\[
\frac{R_i^{(+)}}{R_i^{(-)}} \approx \frac{1 + 2 \langle \mathbf{r}^* \rangle \langle \mathbf{k}^*/k^* \rangle^{(+)} / a}{1 + 2 \langle \mathbf{r}^* \rangle \langle \mathbf{k}^*/k^* \rangle^{(-)} / a} \approx 1 + 2 \langle \mathbf{r}^* \rangle / a
\]

TABLE I. The mean values of spatial and temporal shifts (in fm) of pion and proton sources for two different orientations of the reaction plane in the center of mass frame of the colliding nuclei (upper half) and in the pair rest frame (lower half).

<table>
<thead>
<tr>
<th></th>
<th>( \langle x_\pi - x_p \rangle )</th>
<th>( \langle y_\pi - y_p \rangle )</th>
<th>( \langle z_\pi - z_p \rangle )</th>
<th>( \langle t_\pi - t_p \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi_r = 0 )</td>
<td>-4.7</td>
<td>0.1</td>
<td>-8.3</td>
<td>-3.7</td>
</tr>
<tr>
<td>( \Psi_r = \pi )</td>
<td>1.5</td>
<td>0.1</td>
<td>-7.1</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \langle x^{<em>} - x^{</em>}_p \rangle )</th>
<th>( \langle y^{<em>} - y^{</em>}_p \rangle )</th>
<th>( \langle z^{<em>} - z^{</em>}_p \rangle )</th>
<th>( \langle t^{<em>} - t^{</em>}_p \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi_r = 0 )</td>
<td>-5.8</td>
<td>0.1</td>
<td>-12.3</td>
<td>10.3</td>
</tr>
<tr>
<td>( \Psi_r = \pi )</td>
<td>0.9</td>
<td>0.2</td>
<td>-10.5</td>
<td>8.3</td>
</tr>
</tbody>
</table>

S.V., R. Lednicky, S. Panitkin, Nu Xu, PRL 79 (1997) 4766
2-particle correlations wrt RP

\[ \frac{d^2N}{dx_1dx_2}(x_1, x_2, \Psi_{RP}) \]  
\( x \) - azimuthal angle, transverse momentum, rapidity, etc.

**Approach:**  
- "remove" flow contribution  
- parameterize the shape of what is left  
- study RP orientation dependence of the parameters

Selection of one (or both) of particles in- or out-of the reaction plane "distorts" the RP determination

\[ \frac{dN_{\text{flow}}}{d\Delta\phi_{a,b}} \propto 1 + 2v_{2,b}v_{2,a}^{\text{in, out}} \cos(2\Delta\phi_{a,b}) \]

"a" == "trigger particle"

\[ v_{2}^{\text{in}} = \frac{\pi}{\pi + 4v_{2}} \quad v_{2}^{\text{out}} = \frac{\pi v_{2} - 2}{\pi - 4v_{2}} \]

**CERES, PRL 92(2004)032901**

FIG. 5: In-plane (a) and out-of-plane (b) two-pion opening angle distributions. Dashed lines are calculated for pure elliptic flow as measured by the EP method and corrected for HBT correlations. Data are for centrality 15-30%, \( p_T \geq 1.2 \text{ GeV/c} \), a cut on \( \Delta\theta \geq 20 \text{ mrad} \), and are efficiency corrected. Observe different ordinates as indicated by arrows.

**STAR, PRL 93 (2004) 252301**
Parity, CP-violation, global polarization...

Observing parity (CP) violation with anisotropic flow techniques

S.V. PRC 62 (2000) 044901

Kharzeev, Pisarski, Tytgat, PRL 81 (1998) 512
Kharzeev, Pisarski, PRD 61 (2000) 111901

Emission of positive (negative) pions could be asymmetric along the system angular momentum

D. Kharzeev, hep-ph/0406125:

Large orbital momentum of the system can be transformed into particle spin momentum → global polarization

"Oriented" DCC

Asakawa, Minakata, Müller, nucl-th/0212070

Negative elliptic flow of neutral pions

S.V. nucl-th:0410089

S.V. PRL 81 (1998) 512
S.V. PRD 61 (2000) 111901
S.V. PRC 70 (2004) 057901
S.V. nucl-th:0410079 . PRL
Liang, Wang, nucl-th:0410079 . PRL
How one would measure flow?
Flow induced correlations

\[ \frac{d^2 n}{d_1 d_2} \propto \int \frac{d \Psi_{RP}}{2\pi} \{1 + \sum_n 2v_n \cos[n(1 - \Psi_{RP})]\} \{1 + \sum_n 2v_n \cos\left(2 - \Psi_{RP}\right)\} = 1 + \sum_n 2v_n^2 \cos[n(1 - \varphi_2)] \Rightarrow \langle \cos^2[n(\varphi_1 - \varphi_2)] \rangle = v_n^2 \]

\[ u = e^{i\phi}; \quad Q = \sum u; \quad Q_n = \sum u^n = \sum e^{in\phi} = |Q_n| e^{in\Psi} = X_n + iY_n \]

\[ \langle X \rangle = M v_1; \quad \langle Y \rangle = 0 \]

\[ r_n = Q_n / N \]

\[ \frac{dP}{r_n dr_n} = \frac{1}{\sigma^2} \exp\left(-\frac{v_n^2 + r_n^2}{2\sigma^2}\right) I_0\left(\frac{r_n v_n}{\sigma^2}\right) \]

E877, PRL 73, 2532 (1994)
\[ Q_n = \sum e^{in\phi}; \quad Q_n = |Q_n|e^{i\Psi_n} = X_n + iY_n \]

\[ \Psi_n - n\text{-th harmonic Event Plane} \]

\[ \langle \cos(\Psi_1^{(i)} - \Psi_1^{(j)}) \rangle = \langle \cos(\Psi_1^{(i)} - \Psi_R) \rangle \langle \cos(\Psi_1^{(j)} - \Psi_R) \rangle \]

\[ \nu_n^{obs} = \langle \cos[n(\Psi_m - \Psi_r)] \rangle \]

\[ v_n = \nu_n^{obs} / \langle \cos[km(\Psi_m - \Psi_r)] \rangle \]

\[ \langle \cos[n(\Psi_m^a - \Psi_r)] \rangle = \sqrt{\langle \cos[n(\Psi_m^a - \Psi_m^b)] \rangle} \]

Distribution of hits in the silicon pad detector wrt RP determined by calorimeters.
\[ Q_n = \sum e^{in\phi}; \quad Q_n = |Q_n| e^{in\Psi_n} = X_n + iY_n \]

\[ \Psi_n - n\text{-th harmonic Event Plane} \]

\[ \langle \cos(\Psi^{(i)}_1 - \Psi^{(j)}_1) \rangle = \langle \cos(\Psi^{(i)}_1 - \Psi_R) \rangle \langle \cos(\Psi^{(j)}_1 - \Psi_R) \rangle \]

\[ v_n^{\text{obs}} = \langle \cos[n(\Psi_m - \Psi_r)] \rangle \]

\[ v_n = v_n^{\text{obs}} / \langle \cos[km(\Psi_m - \Psi_r)] \rangle \]

\[ \langle \cos[n(\Psi^a_m - \Psi_r)] \rangle = \sqrt{\langle \cos[n(\Psi^a_m - \Psi^b_m)] \rangle} \]

Distribution of hits in the silicon pad detector wrt RP determined by calorimeters.

E877, PRC 55 (1997) 1420

School of Collective Dynamics in High Energy Collisions, LBNL, May 19-27, 2005

S.A. Voloshin
End of the ideal world: “Non-flow”, flow fluctuations, ...

\[ \langle u_{n,a} u_{n,b}^* \rangle = \langle \cos \left[ n \left( \phi_a - \phi_b \right) \right] \rangle = v_{n,a} v_{n,b} + \delta_n \]

Flow ↑ ↑“non-flow”

\[ \delta_n \ll v_n^2 \]

“Non-flow” - azimuthal correlations of any other origin except the correlation with respect to the reaction plane. It combines the possible contributions from resonance decay, inter and intra jet correlations, etc.

\[ \langle v_{n,a} v_{n,b} \rangle \neq \langle v_{n,a} \rangle \langle v_{n,b} \rangle \]

Effect of flow fluctuations

An example:

\[ v_2 \propto \varepsilon \rightarrow \sigma_v \propto \sigma_\varepsilon \]

\[ \sigma_\varepsilon^2 = \langle \varepsilon^2 (b) \rangle - \langle \varepsilon (b) \rangle^2 \]

Other possibilities for flow fluctuations: fluctuation in the initial geometry, in multiplicity at the same geometry, etc.

Not perfect azimuthal acceptance → not flat Event Plane distribution. “Flattening of the reaction plane”. Approximate and exact solutions to the problem.
End of the ideal world: “Non-flow”, flow fluctuations, ...

\[ \langle u_{n,a} u_{n,b}^* \rangle = \left\langle \cos \left[ n \left( \phi_a - \phi_b \right) \right] \right\rangle = \nu_{n,a} \nu_{n,b} + \delta_n \]

\[ \delta_n \ll \nu_n^2 \]

Flow ↑

"Non-flow" – azimuthal correlations of any other origin except the correlation with respect to the reaction plane. It combines the possible contributions from resonance decay, inter and intra jet correlations, etc.

\[ \langle \nu_{n,a} \nu_{n,b} \rangle \neq \langle \nu_{n,a} \rangle \langle \nu_{n,b} \rangle \]

Effect of flow fluctuations

An example: \[ \nu_2 \propto \varepsilon \rightarrow \sigma_\nu \propto \sigma_\varepsilon \]

\[ \sigma_\varepsilon^2 = \langle \varepsilon^2(b) \rangle - \langle \varepsilon(b) \rangle^2 \]

Other possibilities for flow fluctuations: fluctuation in the initial geometry, in multiplicity at the same geometry, etc.

Not perfect azimuthal acceptance \( \Rightarrow \) not flat Event Plane distribution. "Flattening of the reaction plane". Approximate and exact solutions to the problem.

Each one by itself presents little problem, but taken at the same time, it is the major problem we fight during the last years.
Part II. Methods and Results

1. Non-flow estimates
   - From the “resolution plot”
   - Azimuthal correlations in pp and AA

2. Multiparticle correlations
   - 4-particle cumulants. Methods.
   - Non-flow and flow fluctuations
   - Mixed harmonics. 3-particle correlations.
   - Distributions in q-vector
   - Detector effects

3. Main results
   - Reaching the hydro limit
   - Mass splitting
   - Constituent quark scaling
   - Elliptic flow at high pt
   - $v_1$ and higher harmonics
   - other

4. Conclusion
Mixed harmonic technique or 3-particle correlations

\[ \frac{dN}{d\varphi} \propto 1 + 2a \sin \varphi \]

\( a > 0 \rightarrow \) preferential emission along the angular momentum

The sign can vary event by event, \( a \sim Q/N_{\pi} \), where \( Q \) is the topological charge, \(|Q|=1,2,\ldots\)

\( \rightarrow \) at \( dN/dy \sim 100 \), \(|a| \sim 1\%\).

projections onto reaction plane

Projections on the direction of angular momentum

All effects non sensitive to the RP cancel out!

Possible systematics: clusters that flow

\[ \langle \cos(\varphi_a - \Psi_2) \cos(\varphi_b - \Psi_2) - \sin(\varphi_a - \Psi_2) \sin(\varphi_b - \Psi_2) \rangle = \]

\[ = \langle \cos(\varphi_a + \varphi_b - 2\Psi_2) \rangle = (v_{1,a}v_{1,b} - a_a a_b)\langle \cos(2\Psi_2 - 2\Psi_{RP}) \rangle \]

And using only one particle instead of the event flow vector

\[ \langle \cos(\varphi_a - \varphi_c) \cos(\varphi_b - \varphi_c) - \sin(\varphi_a - \varphi_c) \sin(\varphi_b - \varphi_c) \rangle = \]

\[ = \langle \cos(\varphi_a + \varphi_b - 2\varphi_c) \rangle = (v_{1,a}v_{1,b} - a_a a_b)v_{2,c} \]

note that for a rapidity region symmetric with respect to the midrapidity \( v_1=0 \)
Anti-flow is developing in more peripheral collisions
"Rich" dependence on the particle type: baryons, antibaryons, mesons

Marcus Bleicher, Horst Stocker
Such absorption corresponds to suppression for inclusive yield in central collisions about factor of 4-5.

<\cos(4\varphi)> would behave quite differently (sign, etc.)