Work done by a spring

From the last lecture:

$$K \equiv \frac{mv^2}{2}$$

$$K - K_0 = W$$

Change in the *kinetic energy* equals to *work* done by the external force

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

- The force exerted by a spring is $F = -kx$ where k is the *spring constant*, and x is the displacement from the equilibrium position of the spring.
- The work done by the spring on an object as the object is moved from a displacement x_1 to a displacement x_2 is:

$$W_s = \int_{x_1}^{x_2} (-kx)\,dx = \frac{1}{2}(kx_1^2 - kx_2^2)$$

- In the special case that x_1 is the equilibrium position of the spring, $x_1=0$ so $W_s = -\frac{1}{2} kx^2$.
- The work done by the spring is *positive* if the object ends up *closer* to the equilibrium position $(x = 0)$ and *negative* if the object ends up *further* away.

Potential Energy

- *Potential energy* (U) is energy associated with the *configuration* of the system.
- This type of energy has the potential to become kinetic energy.

    ex. A brick at a height of 1 m above the ground has potential energy. The brick has the *ability to do work* (energy) as it falls to the surface.

Elastic Potential Energy

- Recall that the work done by a spring is: $W_{spring} = \frac{1}{2} k (x_i^2 - x_f^2)$

- Since this depends only on the initial and final positions of the spring, and not on the path taken, we can use this work to define the Elastic Potential Energy of a spring (the potential energy associated with the spring force).

- Since $\Delta U_{spring} = - W_{spring}$, we find $\Delta U_{spring} = \frac{1}{2} k (x_f^2 - x_i^2)$.

- For springs, we will often *define* $U(x=0) = 0$ so $U(x) = \frac{1}{2} kx^2$.

With this convention, the elastic potential energy of a spring is zero when the spring is not stretched or compressed, but it always positive otherwise.

- This means that a deformed (stretched or compressed) has *positive* potential energy; that is, it can do *positive* work to increase the kinetic energy of the system.

Gravitational potential energy

- As we saw previously, gravity does *negative* work, $W_g = -mgh$, when a mass m is lifted through a vertical distance h.
- If an object of mass m is dropped from a height h, gravity will do positive work $W_g = +mgh$ on the mass.
- If no forces besides gravity are acting on the object as it falls, the kinetic energy of the object will be $K = mgh$ just as it hits the ground (work-kinetic energy theorem).
- The system configuration with the mass m a height h has the potential to become kinetic energy $K = mgh$.

- Note that the work done by gravity depends only on the change in vertical position, not the path taken by the system between the two configurations.
- We can therefore define the gravitation potential energy as: $U_g = mgy$

- From this definition, we see that the net work done *on the system* by gravity is equal to the negative of the change in gravitational potential energy:

$$\Delta U_g = -W_g$$

- **Note** : We can choose $U_g = 0$ to be any height we want. The physical property we can measure, the work done, only depends on *changes* in gravitational potential energy. This is similar to choosing the origin of our coordinate system.

Conservative Forces

- Consider a system changing between two configurations, say C_1 and C_2, with a force acting on the objects in this system. On changing from C_1 to C_2, this force does work (W_{12}, possibly negative or zero). On changing from C_2 to C_1 this force will also do work (W_{21}).

- If $W_{12} = -W_{21}$ for *all possible paths* between configuration C_1 and C_2, the force in the problem is said to be a *conservative* force.

  ex. Gravity is a conservative force because the work done in lifting an object some height *h* depends only on *h*, and not on the path taken.

- Forces which are not conservative ($W_{12} \neq -W_{21}$ for some path between C_1 and C_2) are said to be *non-conservative forces*.

  ex. Friction is a non-conservative force. The work done by friction depends on the path taken between the two points (since $\underline{\boldsymbol{F}}_k$ *always* opposes the direction of motion).

- Because the work done by a conservative force depends only on the initial and final configurations of the system, the work done by a conservative force around any *closed* path is equal to *zero*.

Potential Energy (revisited)

- We can define a potential energy associated with every conservative force by measuring the work done by the conservative force in moving the system between two configurations (note: this work is path independent!).

- If a conservative force does *positive* work the potential energy associated with that force is *reduced*.

- If a conservative force does *negative* work the potential energy associated with this force is *increased*.

- Putting these two statements together, we find that: $\Delta U = -W$, *the work done by a conservative force equals the negative of the change in the potential energy associated with that force*.

- The total potential energy in the system is the sum of the potential energies associated with each conservative force in the systems.

•Since the work done by conservative forces is path independent, we can *define* changes in potential energy in terms of work as $\Delta U = -W$.

•The change in potential energy ΔU depends only on the *initial* and *final* configurations of the system, since W is *path independent*.

•Only *changes* in potential energy are physically meaningful. In solving problems, we will often define some configuration to have $U = 0$, and define all other potential energies *relative* to this configuration.

ex. We will often define the potential energy of objects at the surface of the Earth to be zero.

Computing changes in potential energy

•The definition $\Delta U = -W$ allows us to calculate the change in potential energy between two configurations of the system.

•From here on, we will simply assume that the configuration of the system is determined entirely by the (x,y,z) coordinates of a particle (ex. giving the height above the Earth, or the distance a spring is stretched).

Mechanical Energy

- The sum of the kinetic energy of a system and the potential energy of a system (arising from the configuration of the system with respect to conservative forces) is called the mechanical energy (E_{mech}) of the system, $E_{mech} = K + U$.

- Forces that are non-conservative (the work done by the forces depends on the path taken between the initial and final configuration) cannot define a potential energy.

- The work done by non-conservative forces (ex. friction) will increase the internal energy of the system by raising the temperature (more in Chapter 20).

- Non-conservative forces acting on a system can change E_{mech}.

  ex. A book sliding across a table has kinetic energy. Friction, a non-conservative force acts to slow down and then stop the book. This reduces the mechanical energy of the book (but raises the internal energy).