Summary from the last two lectures

$$K \equiv \frac{mv^2}{2}$$

$$\Delta K = K - K_0 = W$$

Change in the *kinetic energy* equals to *work* done by the external force

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

- The force exerted by a spring is where k is the *spring constant*, and x is the displacement from the equilibrium position of the spring.

$$F = -kx$$

$$U_g = \frac{kx^2}{2}$$

- Gravitational force

$$F_g = -mg$$

$$U_g = mgy$$

- For conservative forces

$$\Delta U_{12} = -\int_{x_1}^{x_2} F dx = -W$$

$$E_{mech} = U + K$$

$$\Delta E_{mech} = \Delta U + \Delta K = -W + \Delta K = 0$$

Relation between force and potential energy

- We defined changes in potential energy in terms of the work done by a conservative force.

- In 1D, using differentials, we then find: dU = -dW = $-F_x(x)$ dx.

- Solving for $F(x)$ formally, this gives:

$$F_x = -\frac{dU}{dx}$$

- Graphically this states that the force is negative the *slope* of the *tangent* to the potential energy versus position curve.

- In 3d

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\mathbf{i}} + \frac{\partial U}{\partial y}\hat{\mathbf{j}} + \frac{\partial U}{\partial z}\hat{\mathbf{k}}\right) = -\mathrm{grad}(U)$$

Turning points

- In conservative systems we can write: $U(x)+K(x) = E_{mec}$ (note that this expresses kinetic energy as a function of particle *position* not *velocity*).
- Potential energy U can be *positive* or *negative* (depending on how we define U=0), but the kinetic energy *must* always be *positive*.
- Since the potential energy of a particle can never be negative $K(x) = 0$ (being the boundary between $K(x)>0$ and $K(x)<0$) is called a *turning point*.
- Physically, as a particle approaches a turning point from the region $K(x) > 0$, it will slow down. $K(x) = 0$ means $v = 0$, so the particle comes to rest at this x. Unless $F(x) = 0$ at that point, the particle will then move back into the region where $K(x) > 0$, hence the name turning point.
- Graphically, we will often plot the total mechanical energy of the system as a horizontal straight line on the same plot as $U(x)$ vs x. In this case, the kinetic energy is the difference between this line and the $U(x)$ curve. Turning points occur when E_{mec} crosses the $U(x)$ curve.
- Note that the locations of the turning points depend on the mechanical energy of the system, as $K(x) = E_{mec} - U(x)$

Equilibrium

• A point x is said to be an *equilibrium* point if $F(x) = 0$ (equivalently $dU/dx = 0$) and $K(x)=0$.

• At object at an *equilibrium* point will remain *stationary*.

• If an object is *displaced* slightly from an equilibrium position it can:

A. Remain stationary at a new equilibrium point (*neutral equilibrium*)

ex. A marble sitting on a horizontal table.

B. Move towards the original equilibrium point (*stable equilibrium*)

ex. A marble at the bottom of a round bowl.

C. Move away from the original equilibrium point (*unstable equilibrium*).

ex. A marble sitting on the very top of a billiard ball.

• Graphically, *neutral* equilibrium corresponds to $U(x)$ being constant in a region around x, *stable* equilibrium corresponds to $U(x)$ having a local minima at x, and *unstable* equilibrium corresponds to $U(x)$ having a local maxima at x.

• Note that we can express the stability of x looking at the second derivative of $U(x)$.

Summary

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