Summary from the previous lecture

- For conservative forces

$$\Delta U_{12} = -\int_{x_1}^{x_2} F dx = -W$$

$$F_x = -\frac{dU}{dx}$$

Examples:

$$F_s = -kx$$

$$F_g = -mg$$

$$U_g = \frac{kx^2}{2}$$

$$U_g = mgy$$

Conservation of the mechanical energy:

$$E_{mech} = U + K$$

$$\Delta E_{mech} = \Delta U + \Delta K = -W + \Delta K = 0$$

is an example of a *conservation law*, the mechanical energy is conserved (constant) at all times.

Conservation of mechanical energy - a powerful tool to use in solving problems

Problem solving tips

1. Are all the forces in the problem conservative? If so, you may be able to use the conservation of mechanical energy to solve the problem.
2. Do you know the sum of the potential plus kinetic energy at any time (either beginning, middle, or end)? Remember, for conservative forces this sum will remain constant
3. Do you need to define a zero of potential energy? Remember that our definition for potential energy in terms of work was defined only for changes in potential energy. You may be able to choose some configuration to have $U = 0$, and simplify the problem (note that this will fix all other values of U).

Total energy. Isolated and non-isolated systems

If energy is not transferred across the boundary of a system, we say the system is *isolated*.

- There are several ways to transfer energy into or out of a system:
  - ***Work***: Outside forces can do work on a system OR the system can do work on the outside.
  - ***Waves***: Mechanical and electromagnetic waves (light, microwaves, radio waves, etc) can transfer energy into or out of a system. (NB we will talk more about waves in later chapters).
  - ***Heat***: Energy can flow into or out of a system in the form of heat. This form of energy is associated with the temperature of a system (chapter 20).
  - ***Mass transfer***: Matter transferred into our out of the system leads to a transfer of energy as well.
  - ***Electricity***: Electrical energy can be transferred into or out of a system. This will be covered in more detail in PHY2180/2185.

- Total energy:

$$E_{\text{total}} = K + U + E_{\text{int}}$$

- For an isolated system the total energy is conserved:

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

Isolated systems with non-conservative forces

- Non-conservative can still do work in the system, which changes the kinetic energy, but this work is not associated with a potential energy, but rather changes the internal energy E_{int}.
- Kinetic friction is a non-conservative force (f_K) that does $W = -f_K d$ work when acting over a distance d.
- Note that the work done by kinetic friction is always *negative*, so that it *reduces* the kinetic energy in a system, and therefore *increases* E_{int} in an isolated system.
- If friction is the only non-conservative force present in an isolated system, then

$$\Delta E_{\text{int}} = f_k d = -W_{friction}$$

- With this in mind, for isolated system in which the only non-conservative force acting is friction, we can write:

$$\boxed{\Delta K + \Delta U = W_{friction}}$$

External forces

• The principle of conservation of *mechanical* energy relies on the assumption that the only forces present are internal to the system, and conservative (independent of path).

• More generally, it is possible for an *external* force to do net work (positive or negative) on the system. This will *change* the mechanical energy of the system.

• Work done by the external force may change the potential energy in the system or the kinetic energy in the system (or both).

• If multiple external forces are acting on a system, their net work is the energy transferred into or out of the system.

• In the absence of frictional forces, the work done by an external force is equal to the change in mechanical energy, $\Sigma W = \Delta E_{mech}$.

• If friction is present, we can add the work done by friction on the left side (ΣW) of the equation, where $W_{friction} = -f_k d$.

Power

- *Power* is the rate at which energy is transferred by a force.
- Power is a scalar quantity, measured in Watts (W) with 1 W= 1 Js^{-1}. Power is also often measured in units of horsepower (hp) with 1 hp = 746 W.
- $P_{avg} = \Delta E/\Delta t$, where ΔE is the energy transferred over a time interval Δt.
- The instantaneous power is then:

$$P = \frac{dE}{dt}$$

- From our previous results on energy conservation, we know that the energy transferred into a system is equal to the work done on the system, so we can also write:

$$P_{avg} = \frac{W}{\Delta t} \qquad P = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{d\left(\vec{F} \cdot \vec{s}\right)}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$