Total energy. Isolated and non-isolated systems

If energy is not transferred across the boundary of a system, we say the system is *isolated*.

- There are several ways to transfer energy into or out of a system:
  - ➢***Work***: Outside forces can do work on a system OR the system can do work on the outside.
  - ➢***Waves***: Mechanical and electromagnetic waves (light, microwaves, radio waves, etc) can transfer energy into or out of a system. (NB we will talk more about waves in later chapters).
  - ➢***Heat***: Energy can flow into or out of a system in the form of heat. This form of energy is associated with the temperature of a system (chapter 20).
  - ➢***Mass transfer***: Matter transferred into our out of the system leads to a transfer of energy as well.
  - ➢***Electricity***: Electrical energy can be transferred into or out of a system. This will be covered in more detail in PHY2180/2185.

- Total energy:

$$\boxed{E_{\text{total}} = K + U + E_{\text{int}}}$$

- For an isolated system the total energy is conserved:

$$\boxed{\Delta K + \Delta U + \Delta E_{\text{int}} = 0}$$

Consequences of Newton's Third Law

- Consider two interacting objects. Recall Newton's 3rd Law: $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

- Using Newton's Second Law this gives $m_1\mathbf{a}_1 + m_2\mathbf{a}_2 = 0$, or: $m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$

- We can rewrite this as:

$$\boxed{\frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2\right) = 0}$$

- It means that the quantity $(m_1\mathbf{v}_1 + m_2\mathbf{v}_2)$ is *constant* in time.
- This argument motivates the idea of the *linear momentum* of a particle $\mathbf{p} = m\mathbf{v}$.
- Linear momentum is a *vector*, having *units* of kg m s^{-1}.

The linear momentum of a particle is in the same direction as its *velocity*.

- The time rate of change of linear momentum of a particle is equal to the net applied force acting on that particle.

$$\boxed{\frac{d\vec{p}}{dt} = m\vec{a} = \vec{F}}$$

- Using this expression, a restatement of Newton's First Law would be: If there is no net force exerted on a particle, the momentum of the particle does not change.

Linear momentum of a system

- The linear momentum of a single particle is $\mathbf{p} = m\mathbf{v}$.
- We define the linear momentum of a *system* of N particles having masses $(m_1,\ldots,m_N)$ and with velocities $(\mathbf{v}_1,\ldots,\mathbf{v}_N)$ as $\mathbf{P} = m_1\mathbf{v}_1+\ldots+m_N\mathbf{v}_N$.
- *Newton's Second Law* for a system of particles then states: $\mathbf{F}_{\mathrm{net}} = \mathrm{d}\mathbf{P}/\mathrm{d}t$, where $\mathbf{F}_{\mathrm{net}}$ is the net *external* force acting on the system.
- If there are *no external* forces acting on the system, the momentum of the system is *constant* (but the momenta of the particles comprising the system can still change, due to *internal* forces in the system).
- That is, if particles in a system interact only with one another (no external forces), then the total momentum is constant:

$$\boxed{\vec{P}_{total} = const.}$$

- Since the conservation of linear momentum is a vector equation, we can rephrase this statement as: *If the component of the net external force is zero along an axis, then the component of linear momentum along that axis is constant*.

Impulse

- Since **F** = d**p**/dt, the change in momentum is:

$$\boxed{\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}(t)\,dt}$$

where **F** is explicitly allowed to vary in time.

- We define the *impulse* **I** as: $$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\,dt$$

so that Δ**p** = **I**, or in component form $\Delta p_x = I_x$, $\Delta p_y = I_y$, and $\Delta p_z = I_z$.

- If we know the *average* force $\mathbf{F}_{avg}$ over a time interval Δt, then $\mathbf{I} = \mathbf{F}_{avg}\Delta t$.

- *Newton's Third Law* tells us that if object B exerts a force (which can be time-dependent) on object A, object A exerts an *equal and opposite* force on object B.

- Therefore, if the force from object B provides an *impulse* **I** to object A, the reaction force from object A provides an equal and opposite *impulse* -**I** to object B.

- Since **I** = Δ**p**, the *change* in *momentum* of object B is *negative* the change of momentum of object A.

Collisions

- In principle, the forces acting during a *collision* between two objects can be very complicated.
- However, if neglect *external* forces acting on the system (taken to be the two objects colliding) *during* the collision, the *momentum* of the system will be the same *before* and *after* the collision.
- Neglecting the external forces is called the *impulse approximation*, and is valid if the *internal* forces are much larger than the *external* forces during the collision.
- Collisions for which the total kinetic energy of the system is the same before and after the collision are called *elastic collisions*.
- Collisions for which the total *kinetic energy* of the system is not the same before and after the collision are called *inelastic collisions*.
- Collisions where the two objects colliding stick together (and so have the same final velocities) are called *perfectly inelastic collisions* (or *completely inelastic collisions*).
- For *all* collisions with no external forces, the initial and final momentum of the system is the same, $\mathbf{P}_i = \mathbf{P}_f$.
- If we consider only two body collisions, this is equivalent to $\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$, or $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$.
- Note that these equations hold for *elastic*, *inelastic* and *perfectly inelastic* collisions.

Perfectly inelastic collisions

- In perfectly inelastic collisions, the final velocities of the objects are identical.
- Therefore, the conservation of momentum equation reduces to:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = \left(m_1 + m_2\right)\vec{v}_f$$

where $\mathbf{v}_f$ is the final velocity of the system.

- In the special case that $\mathbf{v}_{2i} = 0$, this reduces to: $\vec{v}_f = \dfrac{m_1}{m_1 + m_2}\vec{v}_{1i}$

Elastic collisions (1d)

- When two objects collide elastically, the *total* kinetic energy of the system remains constant.
- The kinetic energy of *each* object can however change.
- Consider a collision between a stationary object (m_2 with $v_{2i} = 0$) and an object with mass m_1 and velocity v_{1i}. This can always be accomplished by working in the *frame of reference* of object m_2.
- The momentum of the system is always conserved, so $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$.
- If the collision is elastic, kinetic energy is also conserved so:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

- With some algebra, we obtain the following for v_{1f} and v_{2f}:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases of 1d elastic collisions

If $m_1 = m_2$, then $v_{1f} = 0$ and $v_{2f} = v_{1i}$.

ex. When one billiard ball strikes another billiard ball at rest in a head-on collision (an elastic collision between objects with equal masses), the first billiard ball has final velocity zero, while the second billiard ball moves with the initial speed, and in the same direction, as the first billiard ball.

If $m_1 << m_2$, then $v_{1f} \sim -v_{1i}$ and $v_{2f} \sim 0$.

ex. When a ball is thrown against a wall, it bounces straight back with almost exactly the same speed with which is struck the wall. The wall doesn't move.

If $m_1 >> m_2$, then $v_{1f} \sim v_{1i}$ and $v_{2f} \sim 2v_{1i}$.

ex. For the above example, in the frame initially at rest with the ball, the initial speed of the ball is zero, and the wall approaches the ball with a speed v_{1i}. After the collision, the wall is will moving with a speed v_{1i}, but the ball is now moving in the same direction as the wall with a speed of $2v_{1i}$.