Rotation

•Up to this point, we have considered objects as being simply *points* in space. This is sufficient for discussing the *translation* of objects.

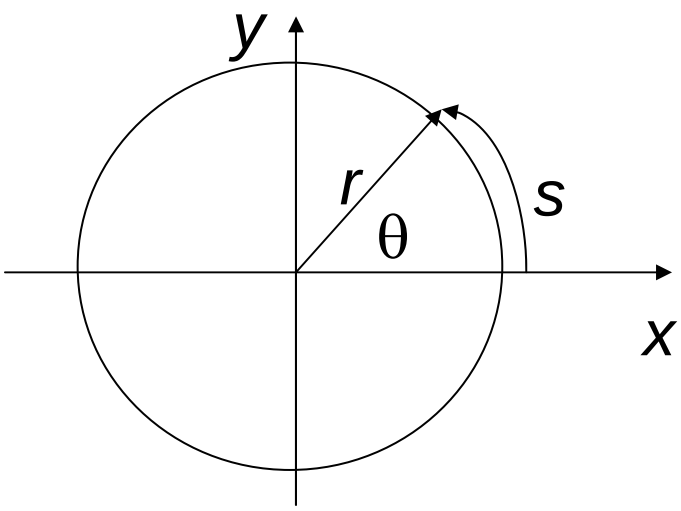
•Other types of motion are also possible for *extended* objects, namely *rotation*.

•We will now consider the rotation of *rigid* objects (the shape of the object does not change) around a *fixed* axis (the rotation occurs around an axis fixed in space).

•In rotational motion, every point of the object moves in a *circular* path around the *axis of rotation*. Moreover, for rigid objects, the *period* of this motion is the same for every point of the object.

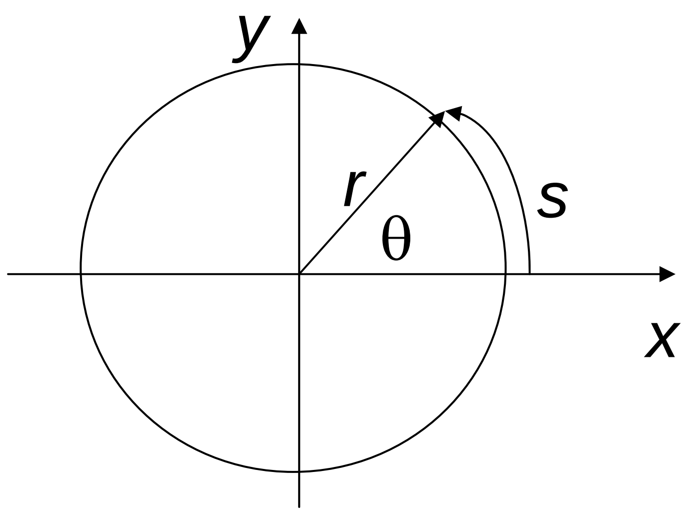
Angular coordinate

- Consider a coordinate system where the x and y axes are *orthogonal* to the axis of rotation.
- In this coordinate system, the x and y coordinates of points in the object will *change* as the object *rotates*; each point of the object will undergo circular motion.
- Since the object is undergoing rigid rotation, the *distance* from the *origin* of the *x*-*y* coordinate system (the axis of rotation) for each point is *fixed*.
- We will therefore describe the rotational motion of the object solely in terms of *angular coordinates*.
- The *angular coordinate* θ is the ratio of the *arc length (s)* along a circular path in the *x*-*y* coordinate system from some reference line (for example, the *x* axis) to the *radius* of this circular path.



Angular displacement

- The angle θ is measured in *radians*, where 2π radians = 360^o
- Note that θ in radians is defined as the *ratio* of two lengths, so it is *dimensionless*.
- As the object rotates, the angular coordinate θ for all points in the object will change, so we can associate the angle θ with the entire rigid object, as well as with individual points in the object.
- We define the *angular displacement* as $\Delta\theta = \theta_2 - \theta_1$.
- Our convention is that a *positive* angular displacement corresponds to a *counterclockwise* rotation, while a *negative* angular displacement corresponds to a *clockwise* rotation.



Angular velocity

• We now define the *average angular velocity* (w_{avg}) of the rotating body over a time interval (t_2 - t_1) as:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

• We define the (instantaneous) angular velocity (ω) as:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

• Note that since the object is assumed to be rotating *rigidly*, the angular displacement of every point of the object is the *same* over the same time interval.

• This means that the angular velocity of every single point of the object is equal to ω, which is simply identified with the angular velocity of the object as a whole.

• Note that ω can be positive or negative. If ω is *positive*, the object is rotating in a *counterclockwise* direction, if ω is *negative*, the object is rotating in a *clockwise* direction.

• The angular velocity ω can vary, leading to an *angular acceleration* α.

Acceleration

- The average angular acceleration is defined as:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

- The instantaneous angular acceleration is defined as:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

θ,ω, and α are *vector* quantities, with both *magnitude* and *direction*. (***Important aside***: strictly speaking, vector addition of angular displacements is only *commutative* when restricted to rotations about the *same rotational axis*).

- The direction associated with these vector quantities is a direction *along* the axis of rotation. This means that points in the rotating object *do not move* at all in this direction.

- The direction associated with the angular velocity vector can be found using the *right hand rule*. With your *right hand*, curl your fingers in the direction of rotation (so points of the object starting by your palm would rotate towards your fingers). Stick your thumb out. Your thumb is pointing in the direction of **ω**.

Motion with a constant acceleration

- Since our derivation of the kinematic equations for linear motion with *constant acceleration* depended only on the *definitions* of velocity as the derivative of displacement, and acceleration as the derivative of velocity, similar equations hold for rotation motion with *constant angular acceleration*.

- With the following *identifications*:

$$\theta \Leftrightarrow x$$
$$\omega \Leftrightarrow v$$
$$\alpha \Leftrightarrow a$$

- We refer to our constant acceleration equations for linear motion to find:

$$\omega = \omega_0 + \alpha t$$
$$\theta - \theta_0 = \tfrac{1}{2}(\omega_0 + \omega)t$$
$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

These equations are only valid for constant angular acceleration α.

Linear and angular coordinates

- We began our discussion of angular coordinates by defining $\theta = s/r$.
- We can use this identification to convert from angular to linear coordinates.
- Define r to be the distance from the axis of rotation to a point of the object.
- Then when the object rotates through an angular displacement θ, this point will describe a circular *arc* of length $s = r\,\theta$.
- Differentiating both sides of this equation with respect to time, and associating ds/dr with the linear speed, we find $v = \omega r$. Note that ω must be *expressed* in radian measure for this equation to be valid.
- It is important to note that while the angular velocity ω is the *same* for every point of the rotating object, the linear speed v depends on the points *distance* from the axis of rotation.
- Since the period of rotation T for an object with linear speed v going around a circle of radius r is $T = 2\pi r/v$, we find immediately that in terms of ω, $T=2\pi/\omega$ or $\omega T=2\pi$.
- Differentiating v=rω with respect to time, with r *constant* and setting dv/dt to be the *tangential* linear acceleration a_t, we find that $a_t=\alpha r$.
- For uniform circular motion $\alpha=0$, so $a_t=0$ but the *centripetal* (or *radial*) acceleration $a_c = v^2/r = \omega^2 r$.