Rotation

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\tau_{net} = I\,\alpha$$

$$I = \int r^2 dm$$

$$K_{rot} = \tfrac{1}{2} I\omega^2$$

$$\tau = F\,s\,\sin\phi$$

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$\omega T = 2\pi$$

$$v = \omega r$$

$$a_c = r\omega^2$$

Rotation $\leftrightarrow$ Translation

$$\theta \Leftrightarrow x$$

$$\omega \Leftrightarrow v$$

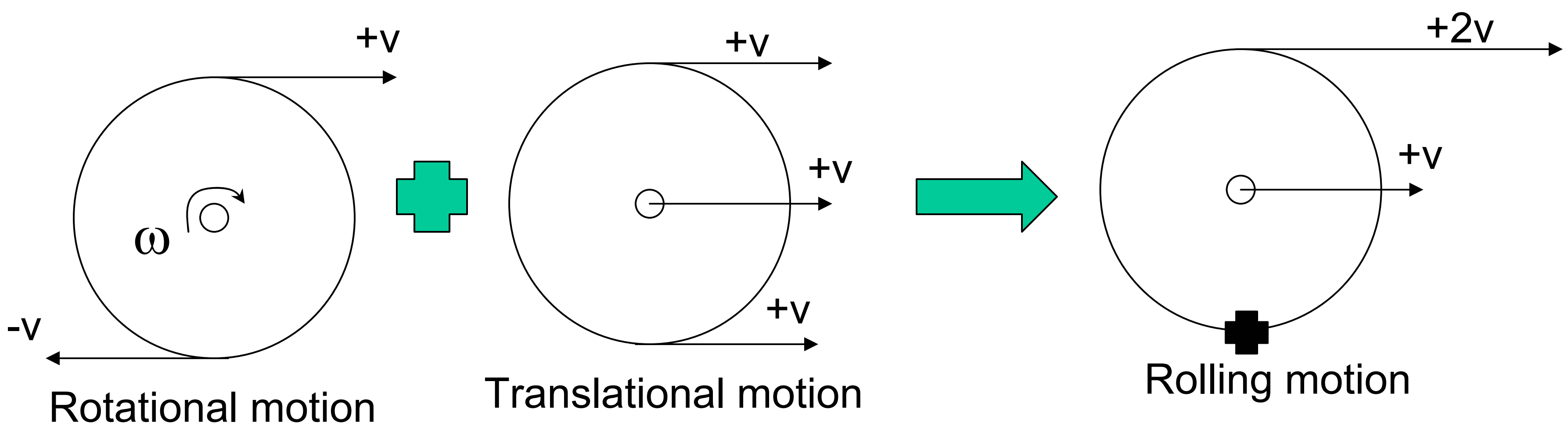
$$\alpha \Leftrightarrow a$$

$$I \Leftrightarrow m$$

$$\tau \Leftrightarrow F$$

Rolling Motion

- When an object is *rolling smoothly* (rolling without *slipping* or *bouncing*) around its center of mass we can consider the motion as a *sum* of a *linear translation* of the center of mass, and a *rotation* of the object around the center of mass.
- $v_{CM} = \omega R$, for a circular object undergoing smooth, rolling motion.
- Instantaneously, the rolling motion of the circle is equivalent to purely rotation motion with axis of rotation at the bottom of the circle (marked by the "+").
- Note that the angular speed of the purely rotational motion will be the same as the angular speed of the rolling motion.



Kinetic energy of rolling

- A rolling object will have contributions to kinetic energy from the translation motion of the CM as well as from the rotation of the object around the CM.
- Considering rolling motion (with rotation around the CM) as being instantaneously equivalent to pure rotation (around the contact point P of the object as discussed previously), we can calculate the total kinetic energy of the object (of mass M).

$$K = \tfrac{1}{2} I_P \omega^2$$

- According to the parallel axis theorem, the moment of inertia through point P is $I_P = I_{COM} + MR^2$.
- Substituting this into the equation for the purely rotational kinetic around P, we find:

$$K = \tfrac{1}{2} I_{CM} \omega^2 + \tfrac{1}{2} MR^2 \omega^2 = \tfrac{1}{2} I_{CM} \omega^2 + \tfrac{1}{2} M v_{CM}^2$$

- since $v_{CM} = \omega R$.
- Note: we have used the fact that the angular speed (ω) of the system is the same for rotation around the COM as for rotation around point P.
- Therefore, a rolling object has a contribution to kinetic energy arising from the linear motion of the COM, and from rotational motion around the COM.

Forces on rolling objects

•Consider a wheel rolling smoothly along a level surface. There is a well-defined relation between the angular and translational components of motion, namely $v_{CM}=\omega R$.

•If v_{CM} changes (by applying a net force to the wheel), ω must change (alternatively, if w changes, v_{CM} must change) if the wheel is to continue rolling smoothly.

•Changing v_{CM} would tend to cause the wheel to slip, unless there is some force acting at the contact point. If the wheel does not slip, this force is the *static frictional force*.

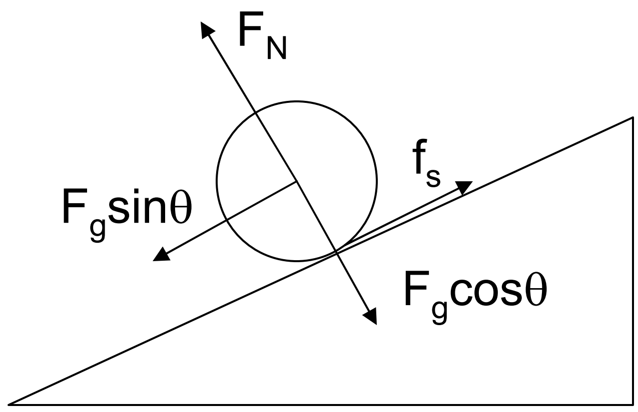
•Differentiating the equation relating v_{CM} to ω with respect to time gives $a_{COM}=\alpha R$ for smooth, rolling motion.

•This is only true if the wheel does not slip as v_{CM} (and therefore ω) changes. If the wheel slips, it is not undergoing smooth, rolling motion, so the relationship between v_{CM} and w no longer holds.

Rolling down a ramp

- Consider a round, uniform object with mass M and radius R rolling smoothly down a rough ($\mu_s \neq 0$) ramp at some angle θ above the horizontal.

- If the object is rolling smoothly down the ramp, we know that $a_{CM} = \alpha R$.



The static frictional force acts *upwards* along the plane (in the absence of friction, the object would slide down the plane) and acts at the contact point between the plane on object.

- With the $+x$ direction up the plane, Newton's Second Law for linear motion tells us $f_s - F_g \sin\theta = m a_{CM}$.

- Taking the center of rotation to be the CM, the only force exerting a torque on the object (since $\mathbf{F}_N \parallel \mathbf{s}$ and F_g acts at the CM so $\mathbf{s}_g = 0$) is f_s.

• The torque exerted by f_s about the CM is $\tau = Rf_s$.

• Newton's Second Law for rotational motion tells us $Rf_s = I_{CM}\alpha$.

• Solving these two equations using $a_{CM} = R\alpha$ (for rolling without slipping) yields:

$$a_{CM} = -\frac{g \sin\theta}{1 + I_{CM}/MR^2}$$

remembering that the negative sign means that a_{CM} is directed *down* the incline plane.

• If the object were to slide down a *frictionless* incline plane, it would accelerate at $a_{CM} = -g\sin\theta$.

• Note that if the object is rolling smoothly down the incline plane, it is not sliding, and therefore the static frictional force does no work.
• This means that for objects rolling smoothly down rough incline planes, mechanical energy will be conserved (and remember that the kinetic energy has contributions from linear as well as rotational motion).