Motion in one dimension

Kinematics is the branch of physics concerned with describing the motion of objects.

•We will first explore the equations of motion for objects constrained to move in one dimension (along a straight line).

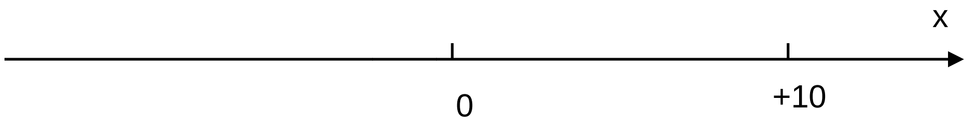
Position

•We locate an object by defining the position x of this object relative to some origin in a given coordinate system.

ex. Given an origin (defining $x = 0$) with a coordinate system defined by positive numbers to the right of the origin, and negative numbers to the left, the position of an object 10 m to the right of the origin would be $x = +10$ m.

ex. With the origin and coordinate system defined above, an object with position $x = -2$ m is located 2 m to the left of the origin

•Note that position has dimension (and units) of *length*, and is a signed quantity.



Displacement

•An object changing location from one position to another position undergoes a **displacement**.

•This **displacement** is defined as the change in position, $\Delta x = x_2 - x_1$. It is independent of the path taken between x_1 and x_2; it depends only on initial and final positions.

•In order for the displacement to be defined, the positions x_1 and x_2 of the object need to be given with respect to the same origin and in the same coordinate system. However, the displacement is independent of the origin.

•The **displacement** is a signed quantity with units of *length*. It is a *vector* having both *magnitude* and *direction*.

ex. An object moves from x_1= +10 m to x_2= -20 m. It's displacement is
Δx= -20m - (+10m) = -30 m.

•**Distance** measures how far the object travels while undergoing a change in position. Distance is a *scalar* quantity, which may or may not be equal to the magnitude of the displacement

Average velocity

•We define **average velocity** as the *displacement* of an object divided by the *time* taken for this displacement.

•Algebraically, $\boldsymbol{v}_{\mathbf{avg}} = \Delta \boldsymbol{x} / \Delta \boldsymbol{t}$.

•The units of *average velocity* are *length*/*time* (m/s in SI units). *Average velocity* is a *vector* quantity with both magnitude and direction.

ex. An object which travels 10 m to the left in a time of 10 s has an average velocity of -1 m/s over those 10 seconds.

•Graphically, the average velocity is the *slope* of the $x(t)$ versus t curve between x_2 and x_1.

•The **average speed** of an object is the *total distance* traveled by the object divided by the *time* taken for this travel.

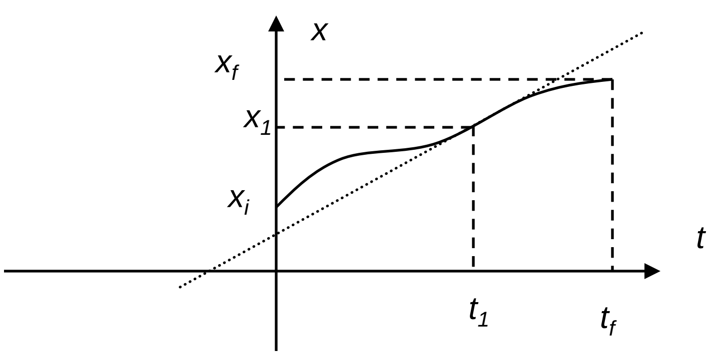
•Average speed is a *scalar* quantity; it has a magnitude but no direction.

Instantaneous velocity

•The **instantaneous velocity** of an object at a time *t* is the rate of change of position of that object at this specific time.
•The **instantaneous velocity** at a time *t* is defined as the average velocity for an infinitesimally small time interval containing *t*.
•Algebraically:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

•Graphically, if one plots the position versus time, the instantaneous velocity at time *t* is the *slope* of the tangent to this curve at *t*.



$$v(t_1) = \frac{dx}{dt}(t_1)$$

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i}$$

•Instantaneous velocity is a *vector* quantity with magnitude and direction.
•**Speed** is defined as the magnitude of instantaneous velocity. It is a *scalar* quantity with magnitude but no direction.

Acceleration

•**Acceleration** is the rate of change of *(instantaneous) velocity*.

•We define the **average acceleration** over some time interval Δt as $a_{avg.} = \Delta v/\Delta t$.

•The **instantaneous acceleration** is the derivative of *velocity* with respect to *time*, or the second derivative of position with respect to time.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

•Graphically, the instantaneous acceleration at time *t* is the slope of the tangent line to the velocity versus time curve at *t*.

•The SI units of acceleration are m/s^2. Accelerations can also be expressed in g units, where 1g is 9.8 m/s^2

ex. An acceleration of 60g is equal to 588 m/s^2.

•Acceleration is a *vector* with both magnitude and direction.

Acceleration, deceleration, and velocity

- Recall that acceleration and velocity are both vector quantities; they have a direction (which in 1-D is simply the *sign*)

- Acceleration and velocity do **not** need to be in the same direction.

- If the velocity and acceleration are in the *same* direction, the magnitude of the velocity is *increasing* (the object speeds up).

- If the velocity and acceleration are in the *opposite* direction, the magnitude of the velocity is *decreasing* (the object slows down).

- Whether an object undergoing an acceleration speeds up or slows down depends on the sign of the acceleration *relative* to the velocity, not simply on the sign of the acceleration.

- Colloquially, it is common to refer to the situation where the velocity and acceleration are in opposite directions as a deceleration. This terminology should be avoided when possible

Constant acceleration

•In many situations, the acceleration of an object can be approximated as being constant.
•The equations of motion have a particularly simple form for constant $a=a_{\text{avg}}$.

$$v_i = v_0;\ \ t_i = 0$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = const \Rightarrow v(t) = v_0 + \int_0^t a\,dt = v_0 + at$$

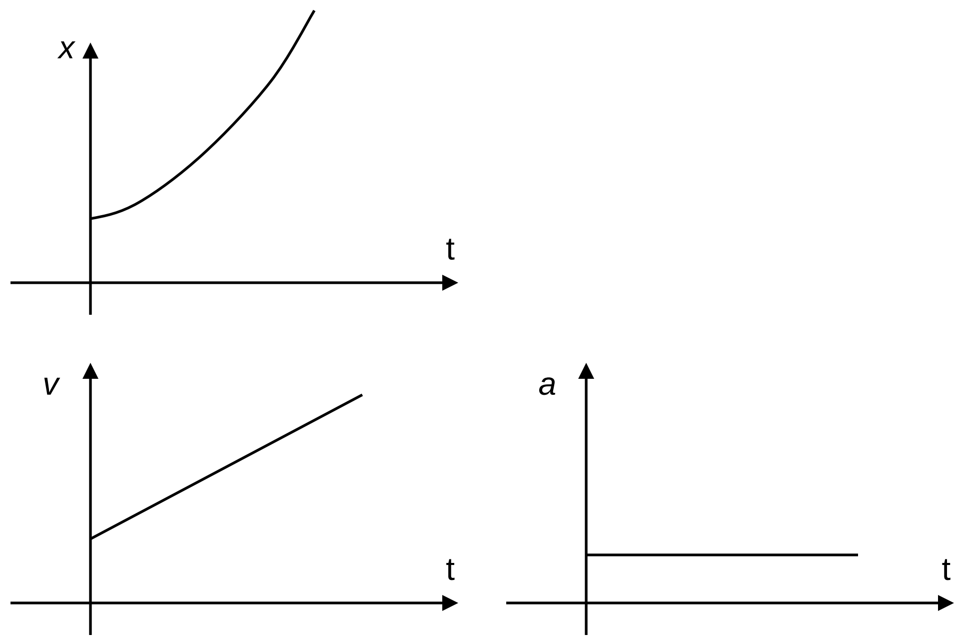
$$x(t) = \int_t^t v(t)\,dt = v_0 t + a\frac{t^2}{2}$$

$$v = v_0 + at$$

$$v_{avg} = \tfrac{1}{2}(v_0 + v)$$

$$x = x_0 + v_{avg}t = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$



•Note that these equations are only valid when *a* is constant. They **cannot** be applied in situations where *a* is not constant.

Free-fall acceleration

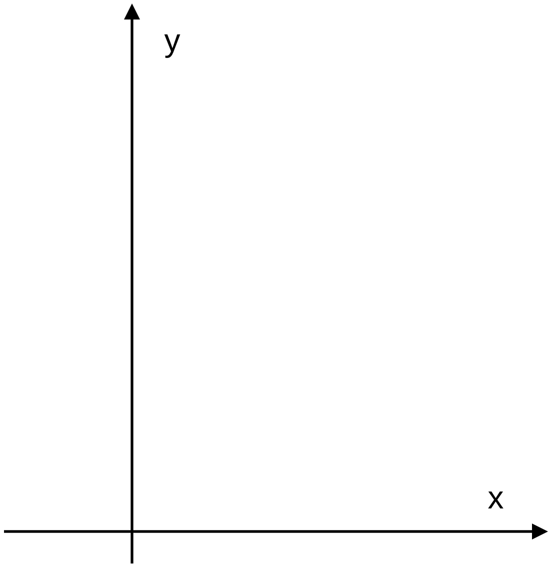
•One very special case of constant acceleration is **free-fall**.

•This acceleration arises from the pull that the Earth's gravitational field exerts on objects.

•For **all** objects near the Earth's surface, the magnitude of this free-fall acceleration is 9.8 m/s^2 towards the center of the Earth. This magnitude is typically denoted by g.

•We will normally choose a coordinate system with the up direction (away from the center of the Earth) being positive and the down direction being negative.

•With this convention, an object in free-fall has a constant acceleration of $a = -g = -9.8$ m/s^2.



Variable acceleration

•If the acceleration is not constant, the kinematic equations discussed previously are not valid

•For the general case of a varying acceleration, we need to use integral and differential calculus to find the equations of motion.

$$a(t) = \frac{dv(t)}{dt} \Leftrightarrow v(t) = \int_{t_i}^{t_f} a(t)dt$$

$$v(t) = \frac{dx(t)}{dt} \Leftrightarrow x(t) = \int v(t)dt$$

$$a(t) = \frac{d^2x(t)}{dt^2} \Leftrightarrow x(t) = \iint a(t)dt$$

•These expressions may be evaluated using graphical techniques when possible