# Equilibrium

- Previously, we discussed that systems with $\mathbf{F}_{net} = 0$ were in equilibrium; the linear acceleration in such systems is zero.
- We now want to extend our discussion of equilibrium to systems for which the angular acceleration is also zero. Using Newton's Second Law for angular motion, this is equivalent to requiring $\boldsymbol{\tau}_{net} = 0$.
- Therefore, for systems in equilibrium we require:

$$\vec{F}_{net} = 0, \;\; \vec{\tau}_{net} = 0$$

- Alternatively, this is equivalent to:

$$\vec{P} = const, \; \vec{L} = const$$

- Recall that the torque (alternatively, angular momentum) of a system is defined relative to a point. The statement $\boldsymbol{\tau}_{net}$=0 (or **L**=constant) means that $\boldsymbol{\tau}_{net}$ must be zero around *every* possible point.
- Note that if $\mathbf{F}_{net}= 0$ *and* $\boldsymbol{\tau}_{net}= 0$ around *any* point, then $\boldsymbol{\tau}_{net}= 0$ around *every* point.

$$\vec{\tau}_O = \vec{r}_1 \times \vec{F}_1 + \ldots + \vec{r}_N \times \vec{F}_N = 0$$

$$\vec{\tau}_{O'} = \left(\vec{r}_1 - \vec{r}\right) \times \vec{F}_1 + \ldots + \left(\vec{r}_N - \vec{r}\right) \times \vec{F}_N = \vec{\tau}_O - \vec{r} \times \left(\vec{F}_1 + \ldots + \vec{F}_N\right) = 0$$

- In other words, if an object is in translational equilibrium, and the net torque about one point is zero, the net torque must be zero about any other point.

# Center of Gravity

- We can consider the force of *gravity* acting on an *extended* body as effectively acting at a single point, called the *center of gravity*.

- If the magnitude of acceleration due to gravity is the *same everywhere* for an extended object, the center of gravity *coincides* with the center of mass.

- This will be true for most objects discussed in this course which are located near the surface of the Earth (where $g=9.8\ ms^{-2}$ is constant).

- The concept of center of gravity is important for calculating torques, since the location of the center of gravity for an object will determine the ***r*** used for calculating $\boldsymbol{\tau}_g = \mathbf{r} \times \mathbf{F}_g$.

# Solving Equilibrium Problems

1. Draw a diagram showing the (extended) objects. Indicate lengths/angles where known.

2. Add arrows representing the forces at the appropriate points on the diagram. Remember that gravity acts at the center of gravity (which is normally the center of mass).

3. Solve for $\mathbf{F}_{net}=0$. It will often be necessary to solve $F_{x,net}=0$, $F_{y,net}=0$, and $F_{z,net}=0$ separately.

4. Solve for $\boldsymbol{\tau}_{net}=0$. Remember that you need to choose some point (or axis) about which to calculate the torque. In general, it is useful to choose a point with a lot of forces going through it, so that the contribution to the torque from these forces is zero. It will often be necessary to solve $\tau_{x,net}=0$, $\tau_{y,net}=0$, $\tau_{z,net}=0$ separately.

5. Steps 3 and 4 may need to be done simultaneously.

# Elasticity

- In many solids, the atoms are arranged in a perfectly periodic, regular *lattice* structure (crystalline solids).

- In such systems, the atoms have a preferred location, and displacing the atoms from this preferred location will lead to an elastic force moving them back.

- Even solids without a well-defined lattice structure (amorphous solids), there are elastic forces holding the atoms in place.

- These elastic forces are what give solids their rigidity.

- As a consequence of these inter-atomic forces, all rigid bodies are somewhat elastic: they can be deformed by externally applied forces.

  e.g. All solids can be stretched or compressed by external forces.

- If the deformation is sufficiently small (below the *elastic limit*) the object will return to its initial shape when the external forces are removed.

- For larger deformations, the object will not recover its initial shape.

# Stress and strain

•We refer to external forces that cause these deformation as *stress*, and the resultant deformation as a *strain*.

•There are many types of stress associated with different types of deformation.

•Stress and strain are related by: *stress*=(*strain*)x(*modulus*), where the *modulus of elasticity* of an object is a measure of its response to external forces.

## Tension and Compression

•Rigid objects will deform if they compressed or pulled on.

•The elastic modulus for this type of deformation is called Young's Modulus (Y) and is defined by:

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

where $F$ is the magnitude of the force applied *perpendicular* to the area $A$ on the object, and $\Delta L/L$ is the relative change in length of the object along the direction of F.

# Shearing

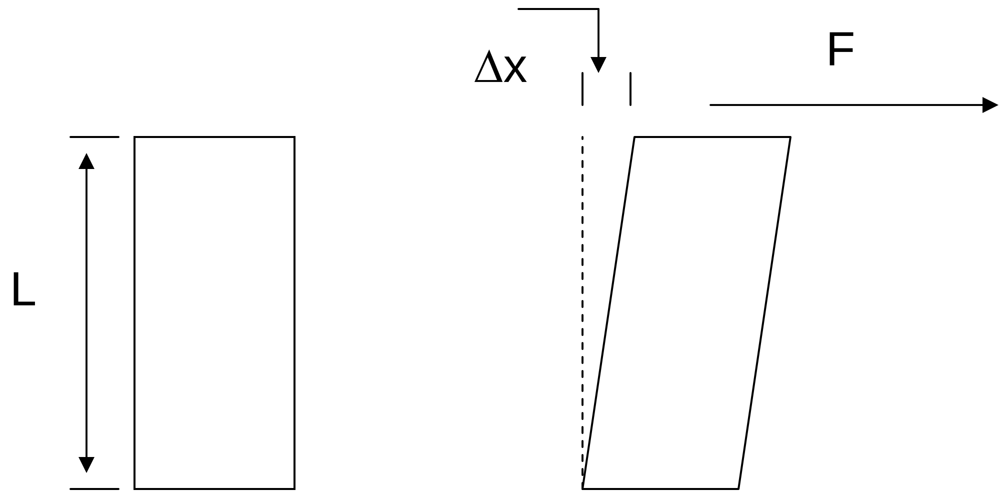
- If the applied force lies in the plane of the area rather than perpendicular to it, the object will deform by shearing.

- The shear modulus (S) of an object is defined by:

$$\frac{F}{A} = S\,\frac{\Delta x}{L}$$

where F is the magnitude of the force *parallel* to A, L is the length of the object perpendicular to A, and $\Delta x$ is the distortion of the object parallel to F.

- Note that fluids cannot support a shear deformation.



# Hydraulic Stress

•If we consider immersing an object in a fluid exerting a large pressure, the object will deform by *compression*.

•The *elastic modulus* corresponding to this type of deformation is called the *bulk modulus* (B).

•The *pressure* on an object is given by P = F/A. If the pressure changes by an amount ΔP = ΔF/A, the volume of the object will change by an amount ΔV, which depends on the bulk modulus.

•Algebraically:

$$B = \frac{-\Delta F / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V}$$

•The negative sign arises because a positive change in pressure gives a negative change in volume.

•Note that liquids have a non-zero bulk modulus, meaning they are difficult to compress.