Mechanical waves

•In many physical systems, the motion of different parts are *coupled* together (exert forces on one another). This is true for rigid bodies (like steel rods) but also for deformable systems (like water or air).

•In these systems, a displacement in one part of the system leads (eventually) to a displacement in another part of the system.

•This displacement propagates through a mechanical wave, which requires three elements:

(1) A disturbance (caused by an external force)
(2) A medium (system) containing elements that can be displaced, and
(3) A physical mechanism through which adjacent elements in the medium can affect one another.

•An individual displacement (disturbance) propagates through the system as a *pulse*.

•A periodic disturbance will travel through the system as a *wave* (a periodic series of pulses).

Transverse and Longitudinal Waves

•A *transverse* wave is one where the motion of each particle (or element of material) is *perpendicular* to the propagation direction of the wave.

ex. A slinky is attached to a wall, and stretched horizontally. Moving the free end of the slinky up an down will produce a *transverse* wave.

•Since there are *two* directions perpendicular to the direction of propagation, there will be *two* independent transverse directions of oscillations.

•A *longitudinal* wave is one where the motion of each particle (or element of material) is *parallel* to the propagation direction of the wave.

ex. A slinky is attached to a wall, and resting on a horizontal table. Moving the free end of the slinky towards and away from the wall will produce a *longitudinal* wave.

•There is only *one* longitudinal direction of oscillation.

•Both *longitudinal* and *transverse* waves are *traveling* waves, because they move from one point to another.

(Sinusoidal) Waves

- In discussing SHM we looked at a particle whose position varied *sinusoidally* with time, according to $x(t) = A\cos(\omega t+\phi)$.

- Now consider a *row* of particles, all undergoing SHM at the same frequency ω.

- If all the particles have the *same* phase ϕ, the entire row of particles will oscillate *together*.

- However, if different particles have *different* values of ϕ the motion will be more complex.

- If the value of ϕ *increases uniformly* from one particle to the next, we will see a *wave* travelling along the row of particles.

- Very roughly, a wave can be thought of as some *collective motion* of a system of particles, where there is no *net* displacement of each individual particle.

  ex. Waves in the ocean propagate as individual elements of water move up and down. After the wave passes, the water elements return to their initial positions.

- A *pulse* is basically a wave where each particle undergoes only one period of motion.

Wave Equations

- Consider a *sinusoidal* transverse wave oscillating in the y-direction traveling along the x-direction of a string.

- At each position x, the piece of string at this position will oscillate like $y_x = A\cos(\omega t+\phi_x)$.

- We can alternatively consider $y(x,t)$ for the entire string, and note that $\phi_x = -kx+\phi$ for a sinusoidal wave so that:

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

- As discussed previously, A is the *amplitude* of the wave, and the quantity $(kx-\omega t+\phi)$ is the *phase* of the wave, k is the *angular wave number* (NOT spring constant), ω is the *angular frequency*, and ϕ is the *phase constant*.

- Note that the *phase* of the wave depends on both x and t.

- The wavelength (λ) of the wave is the difference in x between two successive maxima of the wave at the *same time*, so $A\sin(kx-\omega t)=A(k(x+\lambda)-\omega t)$

- This leads to: $$k\lambda = 2\pi$$

- In a similar way, the *period* T is the difference in time between two successive maxima at the *same position*, so $y_m \sin(kx-\omega t) = y_m(kx-\omega(t+T))$, so:

- The frequency f of a wave is defined as $f=1/T$.

Speed of a Traveling Wave

- We still consider a *transverse* wave traveling along a string.

- Sitting at a *fixed position* x on the string, the value of y will oscillate in *time*.

- Similarly, at a *fixed time* t, the value of y will *vary* with x.

- We can consider *simultaneously* changing x and t to keep the value of y *constant*.

  ex. As a pulse propagates along the string, the x *position* of the peak will change with *time*.

- The value of $y(x,t)$ is *constant* as long as $kx-\omega t$ = constant.

- Recall that $v=dx/dt$, so the *velocity* of wave is given by $kv-\omega = 0$.

- Substituting in the values for λ,T, and *f*, we find that the speed of the wave is:

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

- This wave is traveling in the *positive x* direction.

- A *wave equation* of the form $y(x,t) = y_m \sin(kx+\omega t)$ describes a sinusoidal wave traveling in the *negative* x direction with velocity $v = -\omega/k$.

- More generally, we can define waves of arbitrary shape (not necessarily sinusoidal waves) as functions $y(x,t) = h(kx \pm \omega t)$, where h is any function.

- As long as $kx \pm \omega t$ = constant, the value of y is constant, so this expression defines a traveling wave.

- The *waveform* of a wave is simply the *shape* of the wave. For sinusoidal waves, this will be a simple sine function, however for more complicated functions h, the waveform may be more complex.

Speed of a Wave on a String

• We want to find an expression for the *speed* of a wave traveling on a string.

• The quantities we have to work with are the mass per unit length μ with units ML^{-1} and the tension τ (T is reserved for *period*) in the string with units MLT^{-2}.

• We can combine these to get velocity (LT^{-1}) as: $v \propto \sqrt{\tau/\mu}$

• It can be shown (pg 459 in textbook) that the constant of proportionality is 1.

• Therefore, the speed of a wave traveling along a stretched spring depends only on the *mass per unit length* of the string and the *tension* in the string.

• If we consider some time-varying force producing a wave with frequency f traveling down the string, the wavelength of this wave will be given by $\lambda = v/f$.

Reflections at a Boundary

•When a pulse reaches a boundary it will be *reflected* and travel back along the string in the *opposite* direction.

•If the end of the string is *fixed* at the boundary (for example, attached to a wall), the reflected pulse will have an *opposite* sign to the initial pulse, because we need to have a *node* at the reflection point.

•If the end of the string is *free* to move at the boundary (for example, attached to a ring free to slide on a pole), the reflected pulse will have the *same* sign and amplitude as the incident pulse.

•If the boundary represents a point where the propagation speed of the pulse *changes* (for example, a heavy rope attached to a light string) part of the pulse will continue propagating, and part of the pulse will be reflected (at some phase depending on the *difference* between the the *velocities* in the two materials).

•The *transmitted* pulse will continue with the same phase as the incident pulse.

Energy and Power in a Wave

- As transverse wave moving along a stretched string will carry energy along with it.

- Each section of the string (with mass dm=mdx) moves transversely with SHM, with its maximum speed occurring as it passes through y=0.

- As this section of the string oscillates, its *length* will change slightly, leading to a change in *elastic potential energy* stored in the string. The maximum length of this section of string, hence the maximum value of the elastic potential energy will occur when y=0 as well.

- Therefore, the *maximum* total energy of each section occurs when y=0, and it can be shown that the *minimum* total energy occurs when $y=y_m$.

- As the y=0 part of the pulse (carrying the maximum total energy) travels down the string, the energy associated with this segment of the pulse is carried along.

- The *kinetic energy* associated with each section of the string (which moves only in the y-direction) is: $dK = \frac{1}{2}(\mu dx)v_y^2$

- We know that $y(x,t)=y_m\sin(kx-\omega t)$ so that $v_y=dy/dt$ so that:

$$dK = \tfrac{1}{2}(\mu dx)(-\omega y_m)^2 \cos^2(kx-\omega t)$$

- Then to find the *time rate of change* of kinetic energy we formally divide dK by dt, and remembering that dx/dt is the speed v of the wave we obtain:

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx-\omega t)$$

- This quantity oscillates, but we can determine the *average* rate at with kinetic energy is carried by the wave is:

$$\left(\frac{dK}{dt}\right)_{avg} = \tfrac{1}{4}\mu v \omega^2 y_m^2$$

- Since the average elastic and average potential energy in these systems are equal, the *average* rate at which the *total* energy is carried by the wave is *twice* the rate at which kinetic energy is carried by the wave so that:

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2$$

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Wave Equation

- It is possible (see section 16.6 in the textbook) to derive an expression for the motion of a transverse wave on a stretched string starting from Newton's Second Law and using the assumption that the amplitude of the oscillation is small.

- This leads to the *wave equation*, a differential equation governing the motion of the wave:

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{\tau}\frac{\partial^2 y}{\partial t^2}$$

- More generally, we can write the wave equation as: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

where v is the velocity of the wave.

Note on Sinusoidal Traveling Waves

For a wave described by $y(x,t)=y_m\sin(kx-\omega t)$ **both** curves of y vs x and curves of y vs t will look like *sine waves*. Make sure you understand the difference between these two types of curves, and always carefully check the x-axis label of curves when dealing with traveling waves.

Sound Waves

•A *sound wave* is a *longitudinal* (*compressional*) wave traveling through some *medium*.

•A *point source* generates sound waves traveling in all directions. A *wavefront* of a sound wave is a surface where all the oscillations due to the sound wave have the same value, and a *ray* is a line perpendicular to the wavefront that indicates the direction of travel.

•Close to a point source, the wavefronts are approximately *spherical*, and far away from the source, the wavefronts can be approximated as *planes*.

Speed of Sound Waves

•Recall that the *speed* of transverse waves on a stretched string depends on the *tension* τ and *mass per unit length* μ.

•In a similar manner, the speed of sound waves depends on the *bulk modulus* B of the material (characterizing the amount by which the material compresses when under a pressure p) and the *mass per unit volume* ρ.

$$B = -\frac{\Delta p}{\Delta V / V}$$

•The speed of sound in a material with *bulk modulus* B and *density* ρ is given by:

$$v = \sqrt{\frac{B}{\rho}}$$

•Therefore, materials which are more *difficult* to compress (having higher values of B) will tend to have *higher* speeds of sound (ex. diamond has a large sound speed).

•Because of this, the speed of sound depends weakly on temperature.

•Recall that sound waves are *longitudinal*, so the material oscillates in the *same* direction that the sound waves propagate.

•If we call *s* the displacement of the material from the equilibrium position we can write:

$$s(x,t) = s_m \cos(kx - \omega t)$$

•where s_m is the maximum displacement, *k* is the *angular wavenumber*, and ω the *angular frequency*. Note that we are assuming that s_m is much less than the wavelength of the sound λ.

- As the material oscillates by *compression* and *decompression* the *displacement* and *pressure* will vary *sinusoidally*.

- While the *pressure* and *displacement* both vary *sinusoidally*, the *phase difference* between these two quantities is $\pi/2$.

- The variation in *pressure* can be written as:

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$$

which we can relate to the *maximum* displacement of the oscillations as $\Delta p_m = v\rho\omega s_m$.

Intensity

- The *intensity* of a sound wave is defined as the *average rate per unit area that energy is transferred* by the sound wave, $I=P/A$. Here P is the *power* of the sound wave, and A the *area* intercepting the sound wave.

- It can be shown that $I = \frac{1}{2}\rho v \omega^2 s_m^2$

•For a point source, the sound waves are *spherical*. This means that at a distance *r* from the point source, the *surface* area intercepting the waves will be A=4pr^2.

•We can then express the intensity at a distance r from the source as $I(r)=P_s/4\pi r^2$, where P_s is the *power* of the source.

•Therefore, the sound intensity *decreases* with the *square* of the distance from the source; the intensity follows an *inverse square law*.

•It is often useful to express sound intensities using a *logarithmic* scale called the *Decibel Scale*.

•This is because sound intensities audible to the human ear vary over roughly *12 orders of magnitude.*

•The *sound level* β is defined as $$\beta = (10\,dB)\log\frac{I}{I_0}$$

where dB stands for decibel, the unit of sound level, and the reference intensity I_0 is equal to 10^{-12} W/m^2 (about the threshold for hearing).