# Vectors

•Quantities having both magnitude and direction are referred to as **vectors**. We will use the notation ***s*** to denote vector quantities. The magnitude of ***s*** will be written as *s*. Vectors can have physical units (ex SI units).

•Quantities having only magnitude are referred to as **scalars**.

ex. The magnitude of a vector is a scalar quantity.

•We will often represent vectors as arrows. The *length* of the arrow is proportional to the *magnitude* of the vector and the *direction* of the arrow reflects the *direction* of the vector.

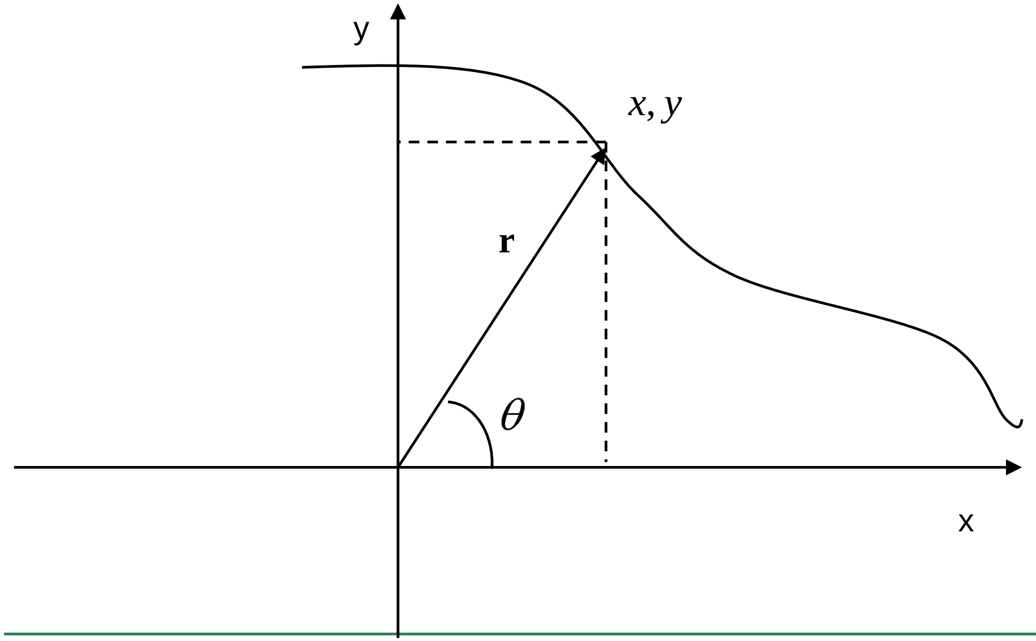
This lecture:

- Coordinate systems
- vector manipulations (addition, subtraction, multiplication by a number).
- basis vectors and vector components

# Rectangular and polar coordinates

•It is convenient to define a *coordinate system* when dealing with a vector.
•This can be used be used to give the direction and length of the vector, both taken relative to the origin of the coordinate system.

•One coordinate system is the rectangular *Cartesian coordinate* system, where mutually orthogonal axes intersect at the origin.
•Another possibility is to define the direction relative to some fixed axis
ex. In *polar coordinates*, we can write **r** = ($r$,θ), where θ is the angle relative to the positive *x*-axis.




$$\mathbf{r} \equiv \vec{r} \equiv (x, y) = (r, \theta)$$

$$r = \sqrt{x^2 + y^2}$$

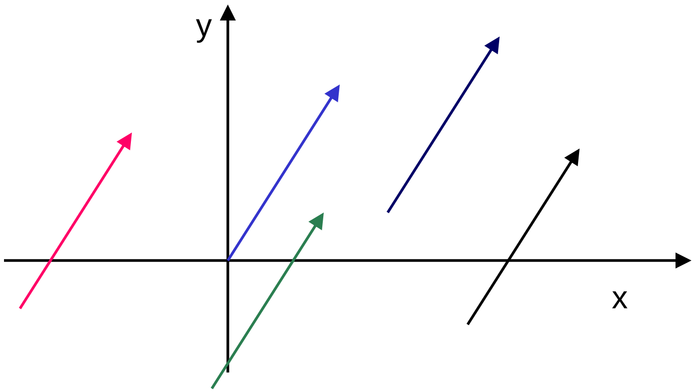
$$x = r\cos(\theta);$$

$$y = r\sin(\theta).$$

# Vector manipulation

•Vectors having *both* the same direction and the same magnitude are *equal*.

NB. This does not depend on what coordinate system is used (ex. a vector expressed in a rectangular coordinate system can be equal to a vector expressed in a polar coordinate system).




•Vectors can be added together *if and only if they represent the same physical quantities*.

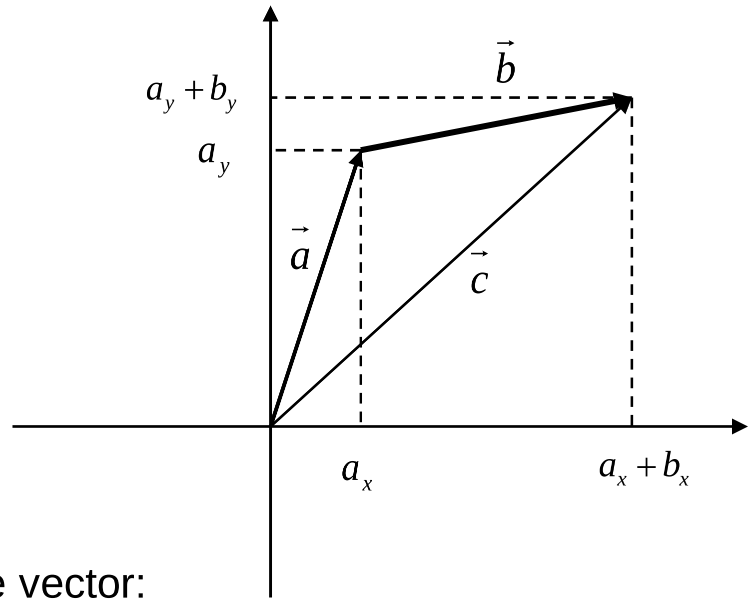
• Vector addition is commutative : $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

and associative: $(\mathbf{a}+\mathbf{b})+\mathbf{c} = \mathbf{a}+(\mathbf{b}+\mathbf{c})$ .

•Graphically, the sum of two vectors can be resolved by drawing the two vectors tip-to-tail. The sum of the two vectors is then represented by the arrow from the tail of the first vector to the tip of the second vector.

NB In the coordinate system picture, this is conceptually equivalent to moving the origin of the coordinate system for the second vector to the position representing the first vector in that coordinate space.

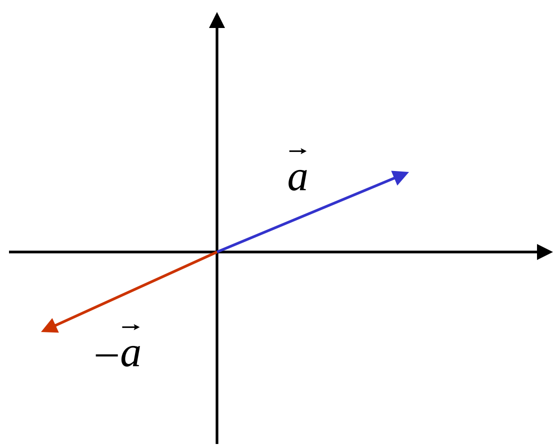
# Vector addition and subtraction




$$\vec{a} + \vec{b} = \vec{c} = (c_x, c_y) = (a_x + b_x, a_y + b_y)$$

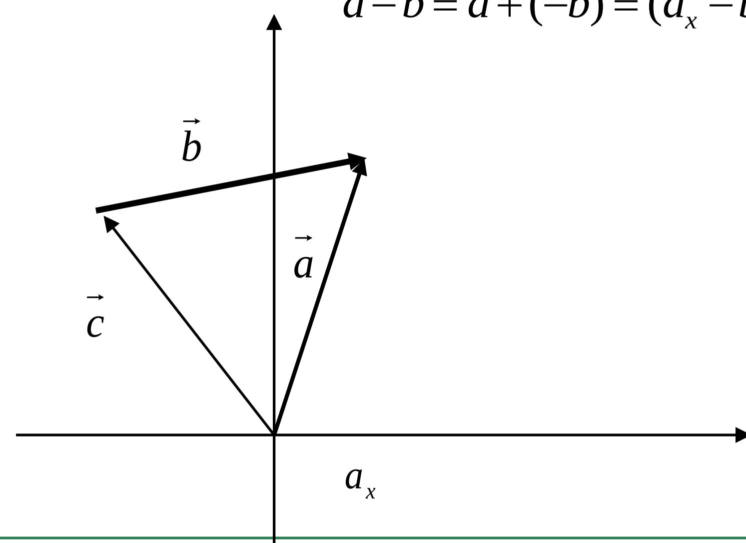
Opposite vector:

$$-\vec{a} \equiv (-a_x, -a_y)$$




Vector subtraction

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = (a_x - b_x, a_y - b_y)$$



# Vector operations

•*Vectors* can be multiplied by *scalars*. The product of a vector ***v*** and a positive scalar *s* (denoted simply *s**v***) is a vector ***y*** having magnitude s*v* pointing in the same direction as ***v***.

•If *s* is negative, *s**v*** is a vector ***y*** with magnitude *sv* pointing in the *opposite* direction of ***v***.

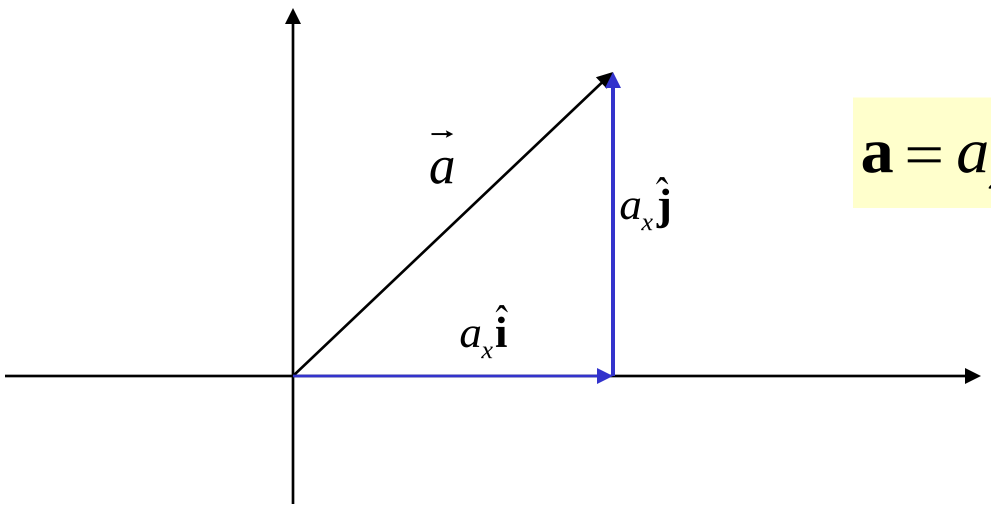
•Graphically, -***a*** is represented as an arrow having the same length as ***a***, but pointing in the opposite direction.

•This allows us to define vector subtraction as ***a***-***b***=***a***+(-***b***).

•Vectors can also be multiplied by other vectors. Depending on how multiplication is defined, the result can be a *scalar* (dot product) or another *vector* (cross product). Dot products and cross products will be discussed later in this course.

# Basis vectors and components

•A vector having magnitude (length) **1** is called a *unit vector*.
•We can define basis vectors: a set of *orthogonal unit vectors* that span all possible directions. For typical *x*-*y* coordinate space, these basis vectors are **i** (pointing in the +*x* direction) and **j** (pointing in the +*y* direction).
•We can then represent a vector **r** by *vector components* $r_x$ and $r_y$, where $r_x$ is in the same direction as *x* and $r_y$ is in the same direction as *y*.
•Since **i** and **j** are unit vectors, we can write this as $\mathbf{r} = r_x\,\mathbf{i} + r_y\,\mathbf{j} = (r_x, r_y)$
•NB $r_x = r\cos\theta$, $r_y = r\sin\theta$ where θ is the angle between *x*-axis and **r**, measured in a counter-clockwise direction.




$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$