Fluids

• A *fluid* is anything that can *flow*. This is equivalent to saying that a fluid is a substance that *cannot support a shear stress*.

• Two important properties in characterizing a fluid are its *density* and *pressure*.

• The *density* of a uniform fluid is its *mass per unit volume*, $\rho = m/V$.

• *Pressure* is defined as the *force per unit area*, $P = F/A$, normally measured in *Pascals* (Pa) which is the same as N/m^2.

Hydrostatic Pressure

- Consider *water* at rest which is in *static equilibrium*.

- Near the *surface* of the water, the pressure in the water must be equal to the pressure of the atmosphere (P_0=101 kPa) because the water is in *equilibrium*.

- Now consider some small volume of water at a *depth h below* the surface, having cross-sectional area *A* perpendicular to the direction of *gravity*.

- The *downwards force* acting on this volume of water is $|F_{down}| = P_0 A + (Ah)\rho g$, where ρ is the density of water.

- Therefore, for this volume of water to be in *equilibrium* $|F_{up}| = P_0 A + (Ah)\rho g$, but F_{up} is simply pA.

- This tells us that at a depth *h*, the pressure of the water is $P(h) = P_0 + \rho g h$, which depends only on *depth*, and not on any *horizontal* dimension of the water.

Pascal's Principle

• *Pascal's Principle* states that a *change* in pressure applied to a *confined, incompressible fluid* is transmitted to every portion of the fluid.

• This is basically an extension of our equation for hydrostatic pressure to $P = P_{ext} + \rho g h$, where P_{ext} is now the *external* pressure acting on the fluid (not necessarily P_0), so that $\Delta P = \Delta P_{ext}$ *everywhere* in the fluid (since we assume that h doesn't change).

• *Pascal's Principle* can be used to make *hydraulic levers*.

• In a *hydraulic lever*, a force F_i is applied to a *piston* of area A_i, which compresses a fluid, changing the *pressure* by an amount F_i/A_i.

• Pascal's Principle states that the pressure everywhere in the fluid changes by an amount F_i/A_i. In particular, the fluid pressing on another piston with area A_o will exert an additional force $F_o = F_i (A_o/A_i)$ on this other piston.

• This is how one can use hydraulics to *magnify* forces.

• Note that the *work* done will *not* be magnified.

Measuring Pressure

- The *absolute pressure* of a fluid is the *total* pressure $P = P_0 + \rho g h$, consisting of contributions from the fluid itself ($\rho g h$), but also a contribution from the *external atmospheric pressure*.

- The *gauge pressure* is just the pressure due to the liquid ($\rho g h$) so $P_{gauge} = P_{absolute} - P_0$.

- Consider a tube, *open* at one end and *closed* at the other placed open end down into an open container filled with *mercury*.

- We assume that the *closed* end of the tube is at $P = 0$ (*vacuum*).

- The mercury in the tube will be at some height h higher than the mercury in the open container, where h satisfies $P_0 = \rho g h$ (here ρ is the density of *mercury*).

- The *height difference* of the mercury in the open end and closed end, expressed in mm is numerically equal to P_0 in units of *torr* (101 kPa=760 torr).

Archimedes Principle

• *Archimides Principle* concerns the forces acting on solid objects *floating in*, or *immersed in*, fluids at rest.

• *Archimedes Principle*: When an object is (partially) submerged in a fluid, the fluid exerts an *upwards buoyant force* on the object with magnitude $m_f g$, where m_f is the mass of fluid *displaced* by the object.

• This can be expressed as $B=m_f g$, where B is the *upwards buoyant force* exerted by the fluid on the object.

• The *maximum* buoyant force acting on an object with volume V is simply $B=(\rho_f V)g$, which occurs when the object is *completely* submerged (*displacing* a volume V of the fluid).

• If an object with volume V is *less dense* (density ρ) than the fluid (density ρ_f), it will *float*.

• The gravitational force acting on floating object is $(\rho V)g$, and the total amount of fluid displaced is $(\rho V)/\rho_f$.

Fluids in Motion

•We will make several assumptions about the motion of fluids to simplify our discussion.

1. *Steady flow*. The *velocity* of fluid flow at a certain point is *constant*. The velocity does not change (magnitude or direction) as a function of time, although different locations in the fluid may flow with different velocities.

2. *Incompressible flow*. The fluid has a *uniform density* and cannot be compressed. This is a good approximation for liquids and a bad approximation for gases.

3. *Nonviscous flow*. There are no resistive forces affecting the flow of the fluid.

4. *Irrotational flow*. The motion of a small volume of fluid can be described solely by *translational* motion.

Equation of continuity

• The *equation of continuity* for fluid flow basically states that fluid cannot be created or destroyed---if it is flowing, it has to go somewhere.

• Suppose a fluid flowing in a tube passes by point 1 with cross-sectional area A_1 having velocity v_1 and passes by point 2 with cross-sectional area A_2 with velocity v_2.

• The rate of fluid flow past point 1 ($\Delta V/t = A_1 v_1$) must equal the rate of fluid flow past point 2 ($A_2 v_2$) so that $A_1 v_1 = A_2 v_2$.

• The *equation of continuity* states that the volume flow rate of a fluid in a tube is constant, Av=constant, where A is the cross-sectional area and v the fluid velocity.

Bernoulli's Equation

- *Bernoulli's Equation* is basically an expression of the conservation of energy for a flowing fluid, as:

$$P + \frac{1}{2}\rho v^2 + \rho g y = const.$$

where p is the *pressure* of the fluid at a given point, v is the *velocity* of the fluid at that point and y the height of the fluid at that point (relative to some y=0).

- The first term (p) is related to the work done by the fluid, the second term ($\frac{1}{2}pv^2$) is the kinetic energy of the fluid and the third term (ρgy) is the potential energy of the fluid.

- In the special case that y=constant (the fluid doesn't change *elevation*) this reduces to $p+1/2\rho v^2$=constant.

- This means that if the elevation of the fluid remains constant, the pressure in the fluid depends on the rate of flow.