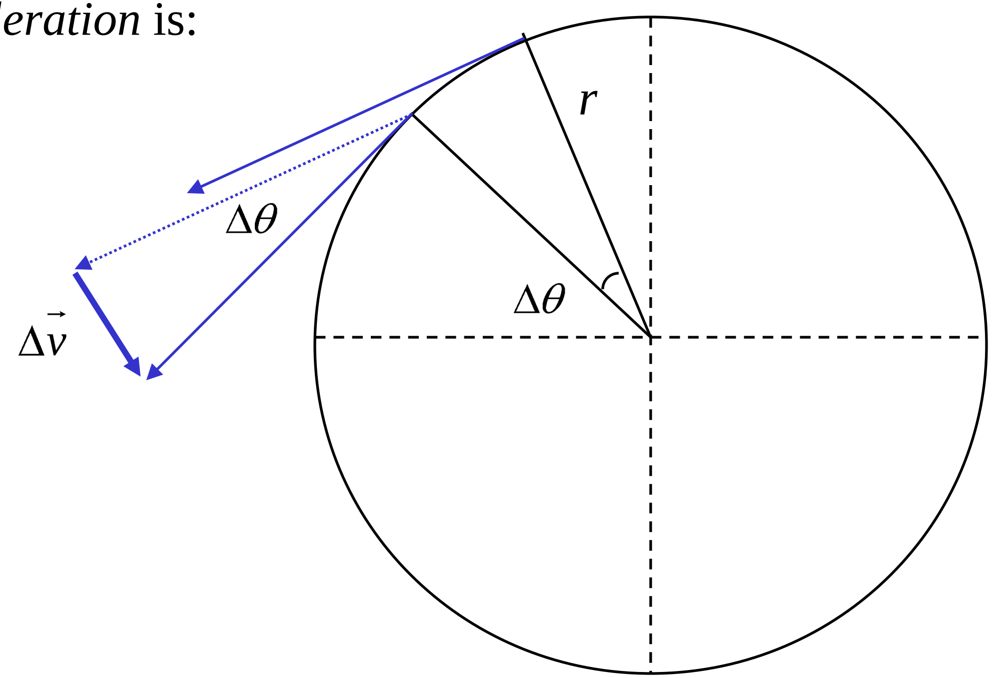
Uniform circular motion

•An object moving at a constant speed around a circular path is said to be undergoing *uniform circular motion*.
•While the magnitude of the instantaneous velocity is constant, the direction of the instantaneous velocity is changing, then the object is *accelerating*.
•At every point along the circular trajectory, the direction of the velocity is tangential to the circular path, and the *acceleration is directed towards the center* of the circle.
•The magnitude of this *centripetal acceleration* is:

$$a_c = \frac{v^2}{r}$$

$$|\Delta\vec{v}| = v\,\Delta\theta; \quad \Delta t = \frac{r\,\Delta\theta}{v};$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



where v is the speed of the object and r the radius of the circular path.
•The time taken for one complete revolution is called the *period* (T) and is given by
$T = 2\pi r/v$

Tangential and radial acceleration

•A particle can also move in a *locally* circular orbit with a non-constant speed.

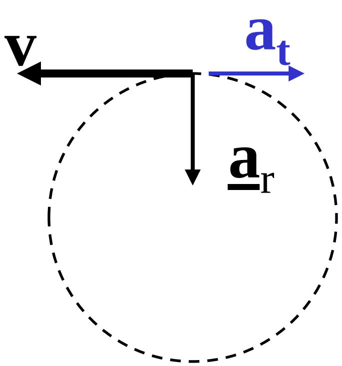
•In this case, the total acceleration of the particle can be expressed as $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, where the radial acceleration $\underline{\mathbf{a}}_r$ arises from the change in velocity direction, while the tangential acceleration $\mathbf{a}_t$ arises from the change in velocity magnitude.

NB By definition $\mathbf{a}_r(t)$ is perpendicular to $\mathbf{v}(t)$, so this cannot change the magnitude of the velocity.

•As the radius of the circular trajectory will, in principle, change with time, we write the radial acceleration as:

$$a_r(t) = -\frac{v^2(t)}{r(t)}$$

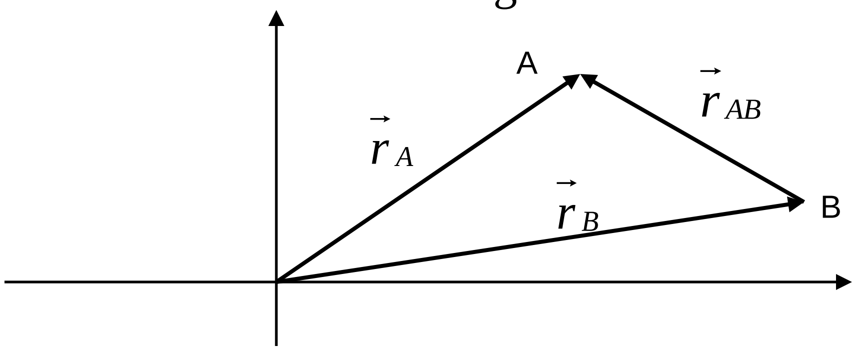
•Note that uniform circular motion has $\mathbf{a}_t = 0$.



Relative motion

•*Relative motion* is a term used to explicitly express the fact that measurements are done by an *observer*, who are in motion themselves.

ex. To an observer riding a train moving at 100 km/h N, the seats on the train are stationary. However, an observer on the ground beside the train would measure the seats moving at 100 km/h N.



$$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$$

•We will define the *reference frame* of an observer to be a coordinate system which is *stationary* with respect to the observer.

•We will define $\mathbf{v}_{AB}$ as the velocity of object A as measured by observer B (or, the velocity of A in reference frame B).

NB Using this definition, $\mathbf{v}_{AA}=0$

•We can use this definition to determine the velocity in different reference frames using the relation $\mathbf{v}_{AC}= \mathbf{v}_{AB}+ \mathbf{v}_{BC}$, or that the velocity of A as measured by C is the velocity of A as measured by B plus the velocity of B as measured by C.

NB Since $\mathbf{v}_{AA}= 0 = \mathbf{v}_{AB}+\mathbf{v}_{BA}$, $\mathbf{v}_{AB}= -\mathbf{v}_{BA}$.

Relative motion (cont'd)

•If two reference frames are moving at constant velocity relative to one another, observers in these two frames will measure the same acceleration for an object P.

•Suppose A and B are moving at constant velocity. Observers in A and B measure the velocity of a particle P as $\mathbf{v}_{PA} = \mathbf{v}_{PB} + \mathbf{v}_{BA}$. Taking the time derivative of both sides, we obtain:

$$\frac{d\vec{v}_{PA}}{dt} = \frac{d\vec{v}_{PB}}{dt} + \frac{d\vec{v}_{BA}}{dt}$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$

where $\mathbf{a}_{PA}$ is the acceleration of P as measured by A, and we have used the fact that $\mathbf{a}_{BA} = 0$ since the frames are moving at constant acceleration relative to one another.