Uniform circular motion

•Recall that the object undergoing *uniform circular motion* (constant speed with a circular trajectory) has a centripetal acceleration $a_c = v^2/R$, directed towards the center of the circular orbit.

•Using Newton's Second Law, $\mathbf{F}_{net} = m\mathbf{a}$, this acceleration must arise from a centripetal force having a magnitude $F_c = mv^2/R$, directed towards the center of the circle.

•If the force directed towards the center of the circle is *not* equal to mv^2/R, the object *cannot* undergo uniform circular motion around a circular path of radius R with constant speed v.

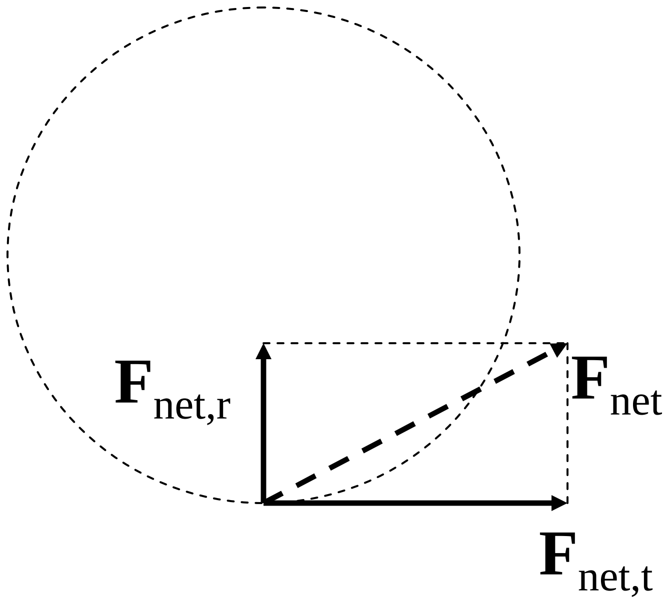
> ex. If you take a sharp corner (small R) very fast (large v) in your car, you will often be "pushed" against the car door. This occurs because the frictional force between you and the seat is insufficiently large to provide the centripetal acceleration to keep you moving with the car. The normal force exerted by the door is necessary to keep you moving with the car.

Non-uniform circular motion

• Recall that if a particle moves with varying speed in a circular path, there is both a radial acceleration ($\underline{\boldsymbol{a}}_r$) and a tangential acceleration ($\mathbf{a}_t$).

• This means that $\mathbf{F}_{net} = m\mathbf{a}_{net} = m(\mathbf{a}_r + \mathbf{a}_t)$.

• We can also express the net force in radial and tangential components as $\mathbf{F}_{net} = \mathbf{F}_{net,r} + \mathbf{F}_{net,t}$.



NB Since $a_c = v^2/r$, if v changes because of a tangential acceleration, either a_c or r (or both) must also change.

Conversely, if a_c changes, either v or r must change as well.

Drag force

• A *fluid* is anything that can flow (deform under shear stresses).

• Any relative velocity between an object and the fluid surrounding it leads to a *drag force* (F_D) being exerted on the object. This force points in the *direction* the fluid flows relative to the object.

ex. A buoy is anchored in a flowing river. The water (a fluid) flowing past the buoy exerts a force F_D on the buoy in the direction of flow.

• Air is a fluid, so all objects moving through the airwill experience a drag force. Equivalently, air moving past a stationary object will exert a drag force on the object (as occurs in a *wind tunnel*).

• In general, determining the magnitude of the drag force is very complicated, as it depends strongly on airflow.

• However, for many situations, we can approximate the magnitude of drag as

$$F_D = \frac{1}{2} D \rho A v^2$$

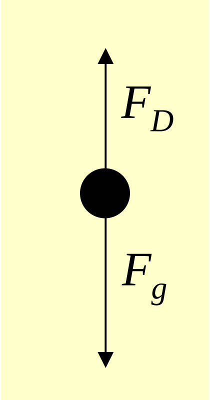
where D is called the *drag coefficient*, ρ is the *fluid density*, and A the *cross-section* of the object perpendicular to **v**.

• For most problems, the magnitude of the drag force is relatively small, and can be neglected. When solving problems only consider the drag force if told explicitly to include it.

Terminal speed

- The terminal speed of the object is the maximum (constant) speed an object reaches in free-fall motion.

- This speed is determined by the requirement that the drag force (acting upwards) on the object is equal in magnitude to the gravitational force (acting downwards).



ex. $\mathbf{F} = m\mathbf{a}$, so $F_D - F_g = ma$. If $F_D = F_g$, $a = 0$

so the object is not accelerating.

- This occurs at a velocity

$$v_t = \sqrt{\frac{2F_g}{D\rho A}}$$

- Note that this depends on the coefficient of drag, D, so different objects (having different drag coefficients) will have different terminal speed.

ex. See Table 6-1 for some terminal speed information.

$y(x)=e^{-2x}$, dy/dx =?

A. $-2x$

B. $2x\ e^{-2x}$

C. $2x\ e^{-2x}$

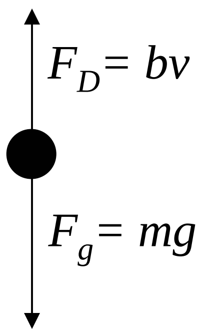
D. e^{-2x}

E. $-2\ e^{-2x}$

Drag Force (II)

•For very small objects (ex. dust in air) or objects moving very slowly, the drag force is found to be proportional to v (rather than the v^2 dependence discussed previously).

•In this case, $\mathbf{F}_D = -b\mathbf{v}$, and we can analytically solve the equation of motion for a falling object.



$$mg - bv = ma = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

Solved by:

$$v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$$