Homework 1, PHY 7500, Fall 2008 (due on September 9)

1. (extra 1 pnt) Find the probability that out of 20 students at least two have their birthdays at the same day.

2. Consider two identical systems with total energy $E = E_1 + E_2 = const$ being in a thermal contact. Assume that the number of (micro)states for each of the subsystems has a power law dependence on its energy, $\Omega_1(E_1) \propto E_1^n$ and $\Omega_2(E_2) \propto E_2^m$. Find the mean and the variance of the distribution in E_1, $\langle E_1 \rangle$ and $\langle (\Delta E_1)^2 \rangle$; compare the exact solution to the one using Gaussian approximation for the number of states.

3. Volume V contains N particles. Assuming no correlation in the location of different particles,
   (a) calculate the probability $p(n, v)$ that the region of volume v contains exactly n particle.
   (b) Show that in the limit $n \ll N$ the distribution assumes the form of Poisson distribution.
   (c) Further, in the limit $1 \ll \langle n \rangle \ll N$ the distribution takes a Gaussian form.
   (d) For all the distributions calculate the mean and the variance.

4. (**1.9**). Making use of the fact that the entropy $S(N, V, E)$ of a thermodynamic system is an extensive quantity ($S(\lambda N, \lambda V, \lambda E) = \lambda S(N, V, E)$), show that

$$N \left(\frac{\partial S}{\partial N} \right)_{V,E} + V \left(\frac{\partial S}{\partial V} \right)_{E,N} + E \left(\frac{\partial S}{\partial E} \right)_{V,N} = S \tag{1}$$

   Note that this result implies: $N\mu = E + PV - TS$.