Homework 3, PHY 7500, due on 09/23/2007

1. Consider two 2-level systems (with energy levels $\pm\varepsilon$), containing N_1 and N_2 particles and with energies $E_1 > 0$ and $E_2 < 0$ being brought in thermal contact. Find the equilibrium values of energies and temperature. Find the fluctuation in energy.

2. Find the distribution of speeds in a relativistic ideal gas of particles of the mass m at temperature T. Examine the non-relativistic limit of your result.

3. Show that in the relativistic ideal gas

$$\langle mv^2/\sqrt{1-(v/c)^2}\rangle = 3kT, \tag{1}$$

where m is the mass of a particle and v its speed. (Hint: Note that $mv^2/\sqrt{1-(v/c)^2} = vp$, the virial). Check that in the extreme relativistic case the thermal energy per particle is twice its value in the non-relativistic case.

4. Calculate the partition function $Q_N(V,T)$ of an extreme relativistic gas consisting of N monatomic molecules with energy-momentum relationship $\varepsilon = pc$, c being the speed of light. Study the thermodynamics of this system, checking in particular that $PV = U/3$, $U/N = 3kT$ and $\gamma = 4/3$. Derive an expression for the density of states $g(\varepsilon)$.