Homework 4, PHY 7500, due on 09/30/2008

1. Consider a system of N noninteracting particles in a container of cross-sectional area A. The bottom of the container is rigid. The top consists of an airtight piston of mass M, which slides without friction. Neglect the potential energy of the molecules of the gas.

   - Construct the partition function Q of the $(N+1)$ particle system (N gas particles of mass m and one piston of mass M).
   - Show that the thermodynamical potential $-kT \ln Q$ is identical to the Gibbs potential of an ideal gas of N molecules.
   - Calculate the fluctuations in the volume of the system.

2. A container with an ideal gas at temperature T has a small hole of area A. Find the distribution in speeds of molecules escaping through the hole and their mean energy. Compare it to the mean energy of the molecules in the gas. If the density of particles in the gas is n, what the rate of energy flow?

3. **3.35 (a)**. Consider a gaseous system of N non-interacting, diatomic molecules, each having an electric dipole moment μ, placed in external electric field of strength E. The energy of a molecule will be given by the kinetic energy of rotation as well as of translation plus the potential energy of orientation in the applied field:

$$\varepsilon = \frac{p^2}{2m} + \left\{ \frac{p_\theta^2}{2I} + \frac{p_\phi^2}{2I \sin^2 \theta} \right\} - \mu E \cos\theta, \tag{1}$$

   where I is the moment of inertia of the molecule. Study the thermodynamics of this system, including the electric polarization and the dielectric constant. Assume that the system is classical and $|\mu E| \ll kT$.