Heavy Flavor Production at RHIC and LHC

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Outline

- Theory of Heavy Flavor Production
- Heavy Flavor Transverse Momentum Distributions
- Electrons from Heavy Flavor Decays
Charm as a Probe of Heavy Ion Collisions

Hard probe produced in the initial nucleon-nucleon collisions
Interacts strongly so its momentum can be modified by collisions during the evolution of the system leading to effects such as

- Energy loss in dense matter (M. Djordjevic et al., Z. Lin et al., D. Kharzeev and Yu. Dokshitzer)
- Transverse momentum broadening due to hadronization from quark-gluon plasma (B. Svetitsky) or cold nuclear matter
- Collective flow of charm quarks (Z. Lin and D. Molnar, R. Rapp et al.)

In addition, if multiple $c \bar{c}$ pairs are produced in a given event, can enhance $J/\psi$ (hidden charm) production (R. Thews et al.)

$pp$ and $d+Au$ collisions serve as an important baseline for understanding medium effects on charm production, need good theoretical background and up-to-date open charm data
Heavy Flavor Measurements

Direct and indirect measurements possible:

- **Direct**: observe flight path and decay vertex, necessary for lifetime measurements
- **Indirect**: observe decay products and, if possible, reconstruct mass

Only reconstruction of parent hadron from decay products can give momentum of heavy flavor hadron, measuring only a decay lepton not enough

Experiments capable of measuring both electromagnetic and hadronic decays can make valuable cross checks

*B* mesons usually measured through $J/\psi$ decay channel
## Charm Hadrons

<table>
<thead>
<tr>
<th>( C )</th>
<th>Mass (GeV)</th>
<th>( c\tau ) (( \mu )m)</th>
<th>( B(C \to lX) ) (%)</th>
<th>( B(C \to \text{Hadrons}) ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^+(cd) )</td>
<td>1.869</td>
<td>315</td>
<td>17.2</td>
<td>( K^-\pi^+\pi^+ ) (9.1)</td>
</tr>
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<td>( D^-(\bar{c}d) )</td>
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<td>315</td>
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</tr>
<tr>
<td>( D^0(c\bar{u}) )</td>
<td>1.864</td>
<td>123.4</td>
<td>6.87</td>
<td>( K^-\pi^+ ) (3.8)</td>
</tr>
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<td>( \bar{D}^0(\bar{c}u) )</td>
<td>1.864</td>
<td>123.4</td>
<td>6.87</td>
<td>( K^+\pi^- ) (3.8)</td>
</tr>
<tr>
<td>( D^{*\pm} )</td>
<td>2.010</td>
<td></td>
<td></td>
<td>( D^0\pi^\pm ) (67.7), ( D^{\pm}\pi^0 ) (30.7)</td>
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<tr>
<td>( D^{*0} )</td>
<td>2.007</td>
<td></td>
<td></td>
<td>( D^0\pi^0 ) (61.9)</td>
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<tr>
<td>( D_s^+(c\bar{s}) )</td>
<td>1.969</td>
<td>147</td>
<td>8</td>
<td>( K^+K^-\pi^+ ) (4.4), ( \pi^+\pi^+\pi^- ) (1.01)</td>
</tr>
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<td>147</td>
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</tr>
<tr>
<td>( \Lambda_c^+(ucd) )</td>
<td>2.285</td>
<td>59.9</td>
<td>4.5</td>
<td>( \Lambda X ) (35), ( pK^-\pi^+ ) (2.8)</td>
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<tr>
<td>( \Sigma_c^{++}(uuc) )</td>
<td>2.452</td>
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<td></td>
<td>( \Lambda_c^+\pi^+ ) (100)</td>
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<tr>
<td>( \Sigma_c^+(ucd) )</td>
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<td>( \Lambda_c^+\pi^0 ) (100)</td>
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<tr>
<td>( \Sigma_c^0(ddc) )</td>
<td>2.452</td>
<td></td>
<td></td>
<td>( \Lambda_c^+\pi^- ) (100)</td>
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<tr>
<td>( \Xi_c^+(usc) )</td>
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<td>132</td>
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<td>( \Sigma^+K^-\pi^+ ) (1.18)</td>
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<tr>
<td>( \Xi_c^0(dsc) )</td>
<td>2.472</td>
<td>29</td>
<td></td>
<td>( \Xi^-\pi^+ ) (seen)</td>
</tr>
</tbody>
</table>

Table 1: Ground state charm hadrons with their mass, decay length (when given) and branching ratios to leptons (when applicable) and some prominent decays to hadrons, preferably to only charged hadrons although such decays are not always available.
Bottom Hadrons

Hadron branching ratios small, two body decays to charged hadrons rare
B decays contribute to lepton spectra in two ways: direct $B \to lX$ and the indirect
chain decay $B \to DX \to lX'$

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<tr>
<td>$B^+(ub)$</td>
<td>5.2790</td>
<td>501</td>
<td>10.2</td>
<td>$\overline{D}^0\pi^-\pi^+\pi^+$ (1.1), $J/\psi K^+$ (0.1)</td>
</tr>
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<td>$B^-(\pi b)$</td>
<td>5.2790</td>
<td>501</td>
<td>10.2</td>
<td>$D^0\pi^-\pi^+\pi^-$ (1.1), $J/\psi K^-$ (0.1)</td>
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<td>$B^0(\bar{d}b)$</td>
<td>5.2794</td>
<td>460</td>
<td>10.5</td>
<td>$D^-\pi^+$ (0.276), $J/\psi K^+\pi^-$ (0.0325)</td>
</tr>
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<td>460</td>
<td>10.5</td>
<td>$D^+\pi^-$ (0.276), $J/\psi K^-\pi^+$ (0.0325)</td>
</tr>
<tr>
<td>$B_s^0$</td>
<td>5.3696</td>
<td>438</td>
<td></td>
<td>$D_s^-\pi^+$ (&lt; 13)</td>
</tr>
<tr>
<td>$B_c^+(c\bar{b})$</td>
<td>6.4</td>
<td></td>
<td></td>
<td>$J/\psi \pi^+$ (0.0082)</td>
</tr>
<tr>
<td>$B_c^-(\bar{c}b)$</td>
<td>6.4</td>
<td></td>
<td></td>
<td>$J/\psi \pi^-$ (0.0082)</td>
</tr>
<tr>
<td>$\Lambda_b^0(udb)$</td>
<td>5.624</td>
<td>368</td>
<td></td>
<td>$J/\psi \Lambda$ (0.047), $\Lambda_c^+\pi^-$ (seen)</td>
</tr>
</tbody>
</table>

Table 2: Known ground state bottom hadrons with their mass, decay length (when given), branching ratios to leptons (when applicable) and some selected decays to hadrons.
Calculating Heavy Flavors in Perturbative QCD

‘Hard’ processes have a large scale in the calculation that makes perturbative QCD applicable: high momentum transfer, $\mu^2$, high mass, $m$, high transverse momentum, $p_T$, since $m \neq 0$, heavy quark production is a ‘hard’ process

Asymptotic freedom assumed to calculate the interactions between two hadrons on the quark/gluon level but the confinement scale determines the probability of finding the interacting parton in the initial hadron

Factorization assumed between the perturbative hard part and the universal, non-perturbative parton distribution functions

Hadronic cross section in an $AB$ collision where $AB = pp, pA$ or nucleus-nucleus is

$$\sigma_{AB}(S, m^2) = \sum_{i,j=q,\bar{q},g} \int_{4m^2/s}^1 \frac{d\tau}{\tau} \int dx_1 dx_2 \delta(x_1 x_2 - \tau) f_i^A(x_1, \mu_F^2) f_j^B(x_2, \mu_F^2) \tilde{\sigma}_{ij}(s, m^2, \mu_F^2, \mu_R^2)$$

$f_i^A$ are the nonperturbative parton distributions, determined from fits to data, $x_1$ and $x_2$ are the fractional momentum of hadrons $A$ and $B$ carried by partons $i$ and $j$, $\tau = s/S$

$\tilde{\sigma}_{ij}(s, m^2, \mu_F^2, \mu_R^2)$ is hard partonic cross section calculable in QCD in powers of $\alpha_s^{2+n}$:

leading order (LO), $n = 0$; next-to-leading order (NLO), $n = 1$ ... 

Results depend strongly on quark mass, $m$, factorization scale, $\mu_F$, in the parton densities and renormalization scale, $\mu_R$, in $\alpha_s$
Calculating the Total Cross Sections

Partonic total cross section only depends on quark mass $m$, not kinematic quantities

To NLO

$$\bar{\sigma}_{ij}(s, m, \mu_F^2, \mu_R^2) = \frac{\alpha_s^2(\mu_R^2)}{m^2} \left\{ f_{ij}^{(0,0)}(\rho) + 4\pi\alpha_s(\mu_R^2) \left[ f_{ij}^{(1,0)}(\rho) + f_{ij}^{(1,1)}(\rho) \ln(\mu_F^2/m^2) \right] + \mathcal{O}(\alpha_s^3) \right\}$$

$\rho = 4m^2/s$, $s$ is partonic center of mass energy squared

$\mu_F$ is factorization scale, separates hard part from nonperturbative part

$\mu_R$ is renormalization scale, scale at which strong coupling constant $\alpha_s$ is evaluated

$\mu_F = \mu_R$ in evaluations of parton densities

$f_{ij}^{(a,b)}$ are dimensionless, $\mu$-independent scaling functions, $a = 0, b = 0$ and $ij = q\bar{q}, gg$

for LO, $a = 1, b = 0, 1$ and $ij = q\bar{q}, gg$ and $qg, \bar{q}g$ for NLO

$f_{ij}^{(0,0)}$ are always positive, $f_{ij}^{(1,b)}$ can be negative also

Note that if $\mu_F^2 = m^2$, $f_{ij}^{(1,1)}$ does not contribute
Scaling Functions to NLO

Near threshold, $\sqrt{S}/2m \to 1$, Born contribution is large but dies away for $\sqrt{S}/2m \to \infty$

At large $\sqrt{S}/2m$, $gg$ channel is dominant, then $qg$

High energy behavior of the cross sections due to phase space and low $x$ behavior of parton densities

![Scaling functions needed to calculate the total partonic $Q\bar{Q}$ cross section. The solid curves are the Born results, $f^{(0,0)}_{ij}$, the dashed and dot-dashed curves are NLO contributions, $f^{(1,1)}_{ij}$ and $f^{(1,0)}_{ij}$ respectively.]

Figure 1: Scaling functions needed to calculate the total partonic $Q\bar{Q}$ cross section. The solid curves are the Born results, $f^{(0,0)}_{ij}$, the dashed and dot-dashed curves are NLO contributions, $f^{(1,1)}_{ij}$ and $f^{(1,0)}_{ij}$ respectively.
Choosing Parameters

Two important parameters: the quark mass $m$ and the scale $\mu$ – at high energies, far from threshold, the low $x$, low $\mu$ behavior of the parton densities determines the charm result, bottom less sensitive to parameter choice

The scale is usually chosen so that $\mu_F = \mu_R$, as in parton density fits, no strict reason for doing so for heavy flavors

Two ways to make predictions:

Fit to Data (RV, Hard Probes Collaboration): fix $m$ and $\mu \equiv \mu_F = \mu_R \geq m$ to data at lower energies and extrapolate to unknown regions – favors lower $m$

Uncertainty Band (Cacciari, Nason and RV): band determined from mass range, $1.3 < m < 1.7$ GeV (charm) and $4.5 < m < 5$ GeV (bottom) with $\mu_F = \mu_R = m$, and range of scales relative to central mass value, $m = 1.5$ GeV (charm) and $4.75$ GeV (bottom): $(\mu_F/m, \mu_R/m) = (1,1), (2,2), (0.5,0.5), (0.5,1), (1,0.5), (1,2), (2,1)$ (Ratio is relative to $m_T$ for distributions)

Need to be careful with $\mu_F \leq m$ and the CTEQ6M parton densities since $\mu_{\text{min}} = 1.3$ GeV, gives big $K$ factors for low scales – problem occurs at low $p_T$

Densities like GRV98 have lower $\mu_{\text{min}}$ so low $x$, low $\mu$ behavior less problematic

Value of two-loop $\alpha_s$ is big for low scales, for $m = 1.5$ GeV:
$\alpha_s(m/2 = 0.75 \text{ GeV}) = 0.648$, $\alpha_s(m = 1.5 \text{ GeV}) = 0.348$ and $\alpha_s(2m = 3 \text{ GeV}) = 0.246$
CTEQ6M Densities at $\mu = m/2$, $m$ and $2m$

CTEQ6M densities extrapolate to $\mu < \mu_{\text{min}} = 1.3$ GeV

When backwards extrapolation leads to $xg(x, \mu) < 0$, then $xg(x, \mu) \equiv 0$

Figure 2: The CTEQ6M parton densities as a function of $x$ for $\mu = m/2$ (left), $\mu = m$ (middle) and $\mu = 2m$ (right) for $m = 1.5$ GeV.
FONLL Calculation (Cacciari and Nason)

Designed to cure large logs of $p_T/m$ for $p_T \gg m$ in fixed order calculation (FO) where mass is no longer only relevant scale

Includes resummed terms (RS) of order $\alpha_s^2(\alpha_s \log(p_T/m))^k$ (leading log – LL) and $\alpha_s^3(\alpha_s \log(p_T/m))^k$ (NLL) while subtracting off fixed order terms retaining only the logarithmic mass dependence (the “massless” limit of fixed order (FOM0)), both calculated in the same renormalization scheme

Scheme change needed in the FO calculation since it treats the heavy flavor as heavy while the RS approach includes the heavy flavor as an active light degree of freedom

Schematically:

$$FONLL = FO + (RS - FOM0) \ G(m, p_T)$$

$G(m, p_T)$ is arbitrary but $G(m, p_T) \rightarrow 1$ as $m/p_T \rightarrow 0$ up to terms suppressed by powers of $m/p_T$

Total cross section similar to but slightly higher than NLO

Problems at high energies away from midrapidity due to small $x$, high $z$ behavior of fragmentation functions in RS result, therefore we don’t calculate results for $|y| > 2$, worse for LHC predictions
Comparison of FONLL and NLO $p_T$ Distributions

FONLL result for bare charm is slightly higher over most of the $p_T$ range – fixed order result gets higher at large $p_T$ due to large $\log(p_T/m)$ terms

New $D^0$ fragmentation functions (dashed) harder than Peterson function (dot-dot-dot-dashed)

Figure 3: The $p_T$ distributions calculated using FONLL are compared to NLO. The dot-dashed curve is the NLO charm quark $p_T$ distribution. The solid, dashed and dot-dot-dot-dashed curves are FONLL results for the charm quark and $D^0$ meson with the updated fragmentation function and the Peterson function, respectively. All the calculations are done with the CTEQ6M parton densities, $m = 1.2$ GeV and $\mu = 2m_T$ in the region $|y| \leq 0.75$. 
Uncertainty Bands for $p_T$ Distributions

Due to range of parameters chosen for uncertainty band, the maximum and minimum result as a function of $p_T$ may not come from a single set of parameters. Thus the upper and lower curves in the band do not represent a single set of $\mu_R$, $\mu_F$ and $m$ values but are the upper and lower limits of mass and scale uncertainties added in quadrature:

$$\frac{d\sigma_{\text{max}}}{dp_T} = \frac{d\sigma_{\text{cent}}}{dp_T} + \sqrt{\left(\frac{d\sigma_{\mu,\text{max}}}{dp_T} - \frac{d\sigma_{\text{cent}}}{dp_T}\right)^2 + \left(\frac{d\sigma_{m,\text{max}}}{dp_T} - \frac{d\sigma_{\text{cent}}}{dp_T}\right)^2}$$

$$\frac{d\sigma_{\text{min}}}{dp_T} = \frac{d\sigma_{\text{cent}}}{dp_T} - \sqrt{\left(\frac{d\sigma_{\mu,\text{min}}}{dp_T} - \frac{d\sigma_{\text{cent}}}{dp_T}\right)^2 + \left(\frac{d\sigma_{m,\text{min}}}{dp_T} - \frac{d\sigma_{\text{cent}}}{dp_T}\right)^2}$$

The central values are $m = 1.5$ GeV (charm) and $4.75$ GeV (bottom), $\mu_F = \mu_R = m_T$.

We follow the same procedure for both the NLO and FONLL calculations and compare them in the central ($|y| \leq 0.75$) and forward ($1.2 < y < 2.2 \ - 1.2 < y < 2$ for FONLL) regions.

Previous (HPC) charm results with $m = 1.2$ GeV, $\mu_F = \mu_R = 2m_T$ fall within the uncertainty band.

Bare heavy quark and heavy flavor meson $p_T$ distributions shown for $pp$ collisions at $\sqrt{S} = 200$ GeV and 5.5 TeV
Components of Uncertainty Band at 200 GeV

Curves with \((\mu_F/m_T, \mu_R/m_T) = (1, 0.5)\) and \((0.5,0.5)\) make up the upper scale uncertainty while those with \((0.5,1)\) and \((2,2)\) make up the lower uncertainty.

Figure 4: The charm quark \(p_T\) distributions calculated using CTEQ6M. The solid red curve is the central value \((\mu_F/m_T, \mu_R/m_T) = (1, 1)\) with \(m = 1.5\) GeV. The green and blue solid curves are \(m = 1.3\) and 1.7 GeV with \((1,1)\) respectively. The red, blue and green dashed curves correspond to \((0.5,0.5)\), \((1,0.5)\) and \((0.5,1)\) respectively while the red, blue and green dotted curves are for \((2,2)\), \((1,2)\) and \((2,1)\) respectively, all for \(m = 1.5\) GeV.
Uncertainty Bands for $c$ and $D$ at 200 GeV

NLO and FONLL bands almost indistinguishable from each other, slight difference in normalization between the two at forward rapidities due to limitations on FONLL at large rapidity.

$D$ meson band uses primary $D$ distributions, not distinguishing charged from neutral $D$ mesons, not possible to separate $c$ and $D$ bands for $p_T < 10$ GeV.

Figure 5: The charm quark theoretical uncertainty band as a function of $p_T$ for FONLL (red solid curves) and NLO (blue dashed curves) in $\sqrt{S} = 200$ GeV $pp$ collisions. Also shown is the $D$ meson uncertainty band (green dot-dashed curves), all using the CTEQ6M parton densities for $|y| \leq 0.75$. 
Comparison to STAR d+Au $D$ Data

Agreement of upper limit of uncertainty band with low $p_T$ STAR data rather reasonable

Figure 6: The FONLL theoretical uncertainty bands for the charm quark and $D$ meson $p_T$ distributions in $pp$ collisions at $\sqrt{S} = 200$ GeV, using $\text{BR}(c \to D) = 1$. Both final and preliminary STAR d+Au data (scaled to $pp$ using $N_{\text{bin}} = 7.5$) at $\sqrt{S_{NN}} = 200$ GeV are also shown.
Components of Uncertainty Band at 5.5 TeV

NLO only here, true FONLL result should be steeper at low $p_T$

Sharp turnover for $(\mu_F/m_T, \mu_R/m_T) = (0.5, 0.5)$ and $(1,0.5)$

Figure 7: The charm quark $p_T$ distributions calculated using CTEQ6M. The solid red curve is the central value $(\mu_F/m_T, \mu_R/m_T) = (1,1)$ with $m = 1.5$ GeV. The green and blue solid curves are $m = 1.3$ and $1.7$ GeV with $(1,1)$ respectively. The red, blue and green dashed curves correspond to $(0.5,0.5), (1,0.5)$ and $(0.5,1)$ respectively while the red, blue and green dotted curves are for $(2,2), (1,2)$ and $(2,1)$ respectively, all for $m = 1.5$ GeV.
Uncertainty Bands for $c$ and $D$ at 5.5 TeV

$c$ and $D$ distributions are harder at 5.5 TeV

Figure 8: The charm quark theoretical uncertainty band as a function of $p_T$ at NLO (red curves) in $\sqrt{s} = 5.5$ TeV $pp$ collisions. Also shown is the $D$ meson uncertainty band (blue curves), all using the CTEQ6M parton densities for $|y| \leq 1$. 

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Comparison of NLO Charm Rapidity Distributions

*pp* distributions broader at 5.5 TeV

Note that the importance of the various mass and scale choices differ considerably between the two energies: faster $Q^2$ evolution at small $x$ with higher scales.

![Diagram showing charm quark rapidity distributions](image)

Figure 9: The charm quark rapidity distributions calculated using CTEQ6M in *pp* collisions at $\sqrt{S} = 200$ GeV (left-hand side) and 5.5 TeV (right-hand side). The solid red curve is the central value $(\mu_F/m_T, \mu_R/m_T) = (1,1)$ with $m = 1.5$ GeV. The green and blue solid curves are $m = 1.3$ and 1.7 GeV with (1,1) respectively. The red, blue and green dashed curves correspond to (0.5,0.5), (1,0.5) and (0.5,1) respectively while the red, blue and green dotted curves are for (2,2), (1,2) and (2,1) respectively, all for $m = 1.5$ GeV.
Uncertainty Bands for $b$ and $B$ at 200 GeV

Bands narrower for bottom than for charm and impossible to separate $b$ from $B$ over the $p_T$ range shown ($B$ is a generic $B$ meson)

Figure 10: The bottom quark theoretical uncertainty band as a function of $p_T$ for FONLL (red solid curves) and NLO (blue dashed curves) in $\sqrt{s} = 200$ GeV $pp$ collisions. Also shown is the $B$ meson uncertainty band (green dot-dashed curves), all using the CTEQ6M parton densities for $|y| \leq 0.75$. 
Uncertainty Bands for $b$ and $B$ at 5.5 TeV

Much stronger energy dependence and more hardening for bottom than for charm with increasing energy.

Figure 11: The bottom quark theoretical uncertainty band as a function of $p_T$ at NLO (red curves) in $\sqrt{S} = 5.5$ TeV $pp$ collisions. Also shown is the $B$ meson uncertainty band (blue curves), all using the CTEQ6M parton densities for $|y| \leq 1$. 
Comparison of NLO Bottom Rapidity Distributions

$pp$ distributions broader at 5.5 TeV

Figure 12: The bottom quark rapidity distributions calculated using CTEQ6M in $pp$ collisions at $\sqrt{s} = 200$ GeV (left-hand side) and 5.5 TeV (right-hand side). The solid red curve is the central value ($\mu_F/m_T, \mu_R/m_T = (1,1)$ with $m = 4.75$ GeV. The green and blue solid curves are $m = 4.5$ and 5 GeV with (1,1) respectively. The red, blue and green dashed curves correspond to (0.5,0.5), (1,0.5) and (0.5,1) respectively while the red, blue and green dotted curves are for (2,2), (1,2) and (2,1) respectively, all for $m = 4.75$ GeV.
Obtaining the Electron Spectra From Heavy Flavor Decays

$D$ and $B$ decays to leptons depends on measured decay spectra and branching ratios.

$D \rightarrow e$ Use preliminary CLEO data on inclusive electrons from semi-leptonic $D$ decays, assume it to be identical for all charm hadrons.

$B \rightarrow e$ Primary $B$ decays to electrons measured by Babar and CLEO, fit data and assume fit to work for all bottom hadrons.

$B \rightarrow D \rightarrow e$ Obtain electron spectrum from convolution of $D \rightarrow e$ spectrum with parton model calculation of $b \rightarrow c$ decay.

Branching ratios are admixtures of charm and bottom hadrons:

\[
B(D \rightarrow e) = 10.3 \pm 1.2\%
\]
\[
B(B \rightarrow e) = 10.86 \pm 0.35\%
\]
\[
B(B \rightarrow D \rightarrow e) = 9.6 \pm 0.6\%
\]
Uncertainty Bands for Electrons from Heavy Flavor Decays at 200 GeV

Electrons from $B$ decays begin to dominate at $p_T \sim 5$ GeV

Electron spectra very sensitive to rapidity range – to get $|y| \leq 0.75$ electrons, need $|y| \leq 2$ charm and bottom range

Forward electron spectra thus not possible to obtain using FONLL code due to problems at large $y$.

![Figure 13: The theoretical FONLL bands for $D \to eX$ (solid), $B \to eX$ (dashed) and $B \to DX \to eX'$ (dot-dashed) as a function of $p_T$ in $\sqrt{s} = 200$ GeV $pp$ collisions for $|y| < 0.75$.](image-url)
Location of $b/c$ Crossover Sensitive to Details of Fragmentation Scheme, Scales, Quark Mass

The $b \rightarrow e$ decays dominate already at lower $p_T$ when standard Peterson function fragmentation ($\epsilon_c = 0.06, \epsilon_b = 0.006$) is used since it hardens charm $p_T$ spectra more than bottom

![Graph showing the ratio of charm to bottom decays to electrons obtained by varying the quark mass and scale factors. The effect of changing the Peterson function parameters from $\epsilon_c = 0.06, \epsilon_b = 0.006$ (lower band) to $\epsilon_c = \epsilon_b = 10^{-5}$ (upper band) is also illustrated. (From M. Djordjevic et al.)](Graph.png)
Comparison to Electron Data at 200 GeV

Includes PHENIX preliminary data from \( pp \) and STAR published and preliminary data.

Figure 15: Prediction of the theoretical uncertainty band of the total electron spectrum from charm and bottom (Cacciari, Nason and RV). Preliminary data from PHENIX and STAR are also shown.
Energy Loss Effects on Single Electrons from 200 GeV Au+Au Interactions

Include radiative energy loss of static system of length $R$ according to DGLV Gluon multiplicities of $dN_g/dy = 1000$ and 3500 used to bracket the PHENIX $\pi^0 R_{AA}$ data

![Graph showing differential cross section](image)

Figure 16: The differential cross section (per nucleon pair) of charm (upper blue) and bottom (upper red) quarks calculated to NLO in QCD compared to single electron distributions calculated with the FONLL fragmentation and decay scheme. The solid, dotted and long dashed curves show the effect of DGLV heavy quark quenching with initial rapidity densities of $dN_g/dy = 0, 1000,$ and 3500, respectively. (From M. Djordjevic et al.)
Comparison of Heavy and Light Quark Energy Loss

At high $p_T$, the charm and light quark $R_{AA}$ coincide because, for both $p_T/m \gg 1$, bottom still too heavy

Figure 17: Heavy quark jet quenching before fragmentation into mesons for $dN_g/dy = 1000$ (left) and 3500 (right) are compared to light ($u, d$) quark and gluon quenching. The resulting $\pi^0 R_{AA}$ is compared to the central 0-10% PHENIX data. (From M. Djordjevic et al.)
Combination of $c$ and $b$ Increases Single Electron $R_{AA}$

Including heavier bottom quark significantly reduces single electron $R_{AA}$ relative to charm alone, $R(e)_{AA} \geq 0.5 - 0.6$ at $p_T \sim 5$ GeV, data much lower.

Figure 18: Single electron attenuation pattern for initial $dN_g/dy = 1000$, left, and $dN_g/dy = 3500$, right. The solid curves employ the FONLL fragmentation scheme and lepton decay parameterizations while the dashed curves use the Peterson function with $\epsilon_c = 0.006$ and $\epsilon_b = 0.006$ and the decay to leptons employed by the PYTHIA Monte Carlo. Note that even for the extreme opacity case on the right the less quenched $b$ quark jets dilute $R_{AA}$ so much that the modification of the combined electron yield from both $c$ and $b$ jets does not fall below $\sim 0.5 - 0.6$ near $p_T \sim 5$ GeV.
How to Reconcile Theory and Experiment?

Data are more similar to $R_{AA}$ for $c \rightarrow e$ alone

How can this be? Some possibilities:

- Mixture of charm and bottom contributions set by pQCD calculations but STAR $D$ $p_T$ distribution is underestimated – relatively larger charm contribution could bring the single electron $R_{AA}$ from $b + c$ down somewhat

- Radiative energy loss only taken into account, more recent results by Djordjevic *et al.* show that previously ignored elastic energy loss could be significant – need to make sure that this doesn’t mess up $R_{AA}$ for light quarks, perhaps reduction in $dN_g/dy$ possible then

- Expansion of system and finite size also need to be accounted for

- Better data would, of course, also be nice...
Uncertainty Bands for Electrons from Heavy Flavor Decays at 5.5 TeV

Crossover between $B$ and $D$ dominance harder to distinguish at LHC energy since $\sqrt{S_{NN}} \gg m_Q$ for charm and bottom.

Electron spectra much harder with increased energy.

Figure 19: The theoretical bands for $D \to eX$ (red curves), $B \to eX$ (blue curves) and $B \to DX \to eX'$ (green curves) as a function of $p_T$ in $\sqrt{S} = 5.5$ TeV $pp$ collisions for $|y| < 1$. 
Summary

- Theoretical uncertainty bands at low $p_T$ show effects of low $x$ and low $\mu$ behavior of parton densities.
- More modern fragmentation functions for $D$ and $B$ mesons indicate that the meson distribution is more similar to the quark distribution to higher $p_T$ than previously assumed from older $e^+e^-$ fits.
- Many open questions remaining regarding single electron $R_{AA}$.
- Contributions of $D$ and $B$ decays to leptons more difficult to disentangle at LHC and would require precision measurements of their decays to hadrons to better distinguish.
- Variety of decay channels needed to sort out results.
Fixing \( m \) and \( \mu^2 \) to All Data: Method 1

Difficult to obtain a large calculated \( c \bar{c} \) cross section with \( \mu_F^2 = \mu_R^2 \), as in parton density fits.

Data favors lower masses – lowest mass used here is 1.2 GeV but much lower masses than allowed in pQCD needed to agree with largest cross sections.

Figure 20: Total \( c \bar{c} \) cross sections in \( pp \) and \( pA \) interactions up to ISR energies as a function of the charm quark mass using the CTEQ6M parton densities. The left-hand plot shows the results with \( \mu_F = \mu_R = m \) while in the right-hand plot \( \mu_F = \mu_R = 2m \). From top to bottom the curves are \( m = 1.2 \) (red), 1.3 (blue), 1.4 (green), 1.5 (magenta), 1.6 (red), 1.7 (blue), and 1.8 (green) GeV.
Extrapolation to Higher Energies

We have kept only the most recent measurements, including the PHENIX $\sqrt{S} = 130$ GeV result from Au+Au, lowest $\sqrt{S} = 200$ GeV point is from PHENIX $pp$

Note the $\mu = m$ behavior at high energy: the cross section grows slower with $\sqrt{S}$ due to the small $x$ behavior of $xg(x, \mu)$ for $\mu$ close to $\mu_{\text{min}}$.

Figure 21: Same as previous but the energy range extended to LHC energies.
Theoretical Uncertainty Band: Method 2

Curves with $\mu_F \leq m$ flatten for $\sqrt{S} > 100$ GeV due to low $x$, low $\mu$ behavior of CTEQ6M – could be different for other PDF sets like GRV98

$(\mu_F/m, \mu_R/m) = (1, 0.5)$ and $(0.5, 0.5)$ have large total cross sections at RHIC since $\alpha_s$ big

Evolution faster at small $x$ and high $\mu$ [(2,2), (2,1)]

$K$ factors bigger with this method
Figure 22: Total $\sigma$ cross sections calculated using CTEQ6M. The solid red curve is the central value $(\mu_F/m, \mu_R/m) = (1,1)$ with $m = 1.5$ GeV. The green and blue solid curves are $m = 1.3$ and 1.7 GeV with (1,1) respectively. The red, blue and green dashed curves correspond to $(0.5,0.5), (0.5,1)$ and $(1,0.5)$ respectively while the red, blue and green dotted curves are for $(2,2), (2,1)$ and $(1,2)$ respectively, all for $m = 1.5$ GeV.
Comparison of Bottom Calculations to Data

Fewer data on bottom production in \( pp \) collisions, especially on total cross section

Bottom production is less problematic because, even for \( \mu = m/2 \), we are well above \( \mu_{\text{min}} \) of parton densities, extrapolation to higher energies should also be better
Fixing $m$ and $\mu^2$ to All Data: Method 1

Latest HERA-B point not shown, lies below previous point

In this approach, $m = 5$ GeV, $\mu = m/2$; $m = 4.75$ GeV, $\mu = m$; and $m = 4.5$ GeV, $\mu = 2m$ are all close to center of data.

Figure 23: Total $b\bar{b}$ cross sections in $pp$ and $pA$ interactions as a function of the bottom quark mass using the CTEQ6M parton densities. Clockwise from upper left, the plots give results for $\mu = m/2$, $\mu = m$ and $\mu = 2m$. The mass values are 4.5 GeV (solid red), 4.75 GeV (dashed blue) and 5 GeV (dot-dashed green).
Extrapolation to Higher Energies

Asymptotic behavior very similar for bottom, no surprises

Figure 24: Same as previous but the energy range extended to LHC energies.
$K$ Factors Using Method 1

$K$ factors better behaved for bottom production, $x$ and $\mu$ not so small as for charm, consequently $\alpha_s$ is smaller also

$K$ factors much smaller at higher energy than charm, strong growth only seen for $\mu = m/2$, smallest $K$ factors for $\mu = 2m$, also the case with charm

Figure 25: The $K$ factors over the full $\sqrt{S}$ range, labeled as before.
Theoretical Uncertainty Band: Method 2

More sensible to talk about uncertainty band for bottom than for charm

Figure 26: Total $b\bar{b}$ cross sections calculated using CTEQ6M. The solid red curve is the central value $(\mu_F/m, \mu_R/m) = (1, 1)$ with $m = 4.75$ GeV. The green and blue solid curves are $m = 4.5$ and 5 GeV with (1,1) respectively. The red, blue and green dashed curves correspond to $(0.5,0.5)$, $(1,0.5)$ and $(0.5,1)$ respectively while the red, blue and green dotted curves are for $(2,2)$, $(1,2)$ and $(2,1)$ respectively, all for $m = 4.75$ GeV.
From Total Cross Sections to Distributions

Distributions as a function of kinematic variables can provide more information than the total cross section.

In total cross section, the quark mass is the only relevant scale.

When considering kinematic observables like $x_F$ or $p_T$, the momentum scale is also relevant so that, instead of $\mu^2 \propto m^2$, one usually uses $\mu^2 \propto m_T^2$ – this difference makes the $p_T$-integrated total cross section decrease a bit relative to that calculated using the dimensionless scaling functions.

Fragmentation also important when discussing observables.

Fragmentation universal, like parton densities, so the parameterizations of $e^+e^-$ data should work in hadroproduction – new determinations of the charm to $D$ fragmentation in Mellin space result in a softer, more accurate spectra than the old Peterson function.
NLO Bare Quark $p_T$ Distributions

Differences largest at low $p_T$, determines total cross section

Distributions become similar at high $p_T$

Average $p_T$ increases with $m$ and decreases with $\mu$

Figure 27: The NLO charm quark $p_T$ distributions in $pp$ interactions at $\sqrt{S} = 200$ GeV as a function of the charm quark mass calculated with the GRV98 HO parton densities, integrated over all rapidity. The left-hand plot shows the results with the renormalization and factorization scales equal to $m_T$ while in the right-hand plot the scale is set to $2m_T$. From top to bottom the curves are $m = 1.2$ (red), 1.3 (blue), 1.4 (green), 1.5 (magenta), 1.6 (red), 1.7 (blue), and 1.8 (green) GeV.
## Charm Cross Sections

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<tr>
<th>$m$ (GeV)</th>
<th>$\mu_F/m_T$</th>
<th>$\mu_R/m_T$</th>
<th>$\sigma$(200 GeV) (mb)</th>
<th>$\sigma$(5.5 TeV) (mb)</th>
<th>$\sigma$(8.8 TeV) (mb)</th>
<th>$\sigma$(14 TeV) (mb)</th>
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Table 3: Charm cross sections obtained from the parameter sets used to determined the theoretical uncertainty band in $pp$ collisions with the CTEQ6M densities.
## Bottom Cross Sections

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<th>$m$ (GeV)</th>
<th>$\mu_F/m_T$</th>
<th>$\mu_R/m_T$</th>
<th>$\sigma(200\text{ GeV})$ (µb)</th>
<th>$\sigma(5.5\text{ TeV})$ (µb)</th>
<th>$\sigma(8.8\text{ TeV})$ (µb)</th>
<th>$\sigma(14\text{ TeV})$ (µb)</th>
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Table 4: Bottom total cross sections obtained from the parameter sets used to determined the theoretical uncertainty band in $pp$ collisions with the CTEQ6M densities.